SOLVING FOR IRRATIONAL ZEROS: WHITENESS IN MATHEMATICS TEACHER EDUCATION

by

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ABSTRACT

For many, mathematics and social justice are perceived as incompatible. Several mathematics education researchers have noted resistance to social justice among mathematics teachers. However, mathematics education has a consistently negative impact on the education of students of color. This study seeks to better understand the nature of this resistance by studying how preservice secondary mathematics teachers grapple with understanding social justice mathematics education. For this study I draw on discursive understandings of the operation of power and Whiteness Theory in order to understand the ways in which the discourses of mathematics serve to exclude the discourses of social justice. The participants in this study were seven preservice secondary mathematics teachers in a master's degree program in mathematics with teaching certification. Class discussions were recorded and transcribed then analyzed using Critical Discourse Analysis and a Whiteness Theory lens to interpret the analysis. The findings are organized around three main themes. These themes include discourses of the abstract nature of school mathematics, teacher and student subject positions, and our struggle to engage with the concepts of social justice mathematics. At times we disrupted these discourses through playfulness, repositioning students, and embracing the struggle of incorporating social justice into mathematics. There are important implications for mathematics education, mathematics teacher education, and teacher education generally.

This dissertation is dedicated to my students at Battle Mountain High School who taught me about being a teacher, and to the seven mathematics teachers who participated in this study.

CONTENTS

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CHAPTER 1

CHALLENGES IN TEACHING MATHEMATICS FOR SOCIAL JUSTICE

Recent research suggests that secondary mathematics teachers have difficulty teaching mathematics for social justice and that when the attempt is made teachers tend to focus either on mathematics or social justice. Mathematics teachers, in particular, have difficulty integrating mathematics and social justice to achieve both mathematical and social justice goals simultaneously (Bartell, 2013; Brantlinger, 2013). Social justice and mathematics, for these teachers, seem to be like oil and water. Even when teachers bring the two together they simply will not stay mixed in the moment of teaching. This kind of divided thinking has led some teachers to give up on or delay efforts to teach mathematics for social justice (Brantlinger, 2013). As a mathematics teacher I have faced some of these same difficulties and felt that the mathematics would not allow me to teach in ways that would meet the needs of my students. I watched my students struggle with the mathematics, especially those learning English, and how disconnected the mathematics was from the things they really cared about. Despite my ability to understand the mathematics well, to teach clearly, and to make connections with my students, the mathematics, at times, seemed to get in the way.

However, there are alternatives to an oil and water perspective on the teaching of mathematics for social justice. A poststructuralist discursive perspective may open up

greater possibilities to understand the difficulties involved in teaching mathematics for social justice and aid teachers in learning to teach mathematics for social justice. In particular an understanding of the ways in which the discourses of mathematics and the discourses of Whiteness intertwine may create new possibilities for mathematics teachers to more effectively teach mathematics for social justice.

Problem Statement

Difficulties in Teaching Mathematics for Social Justice

While teaching for social justice is not an easy task in any field, it may be particularly difficult in mathematics, since the perception of mathematics as abstract, apolitical, and acultural may appear to form an insurmountable divide from the explicitly political and contextualized perception of social justice. One source of this divide between teaching for social justice and mathematics may lie within the way modern mathematics is conceptualized and understood. Modern mathematics is dominated by abstract, decontextualized problems written with formal, symbolic language (Walkerdine, 1988). Modern mathematics is further assumed to transcend the concerns and problems of the world to focus on those things that are universal (de Freitas, 2013). This way of thinking about mathematics has created a way of talking about and enacting mathematics that is focused on the static existence (in this fictional world) of mathematical objects (de Freitas, 2013), rather than on solving meaningful, contextualized problems or on the inclusion of alternative ways of thinking (except for professional mathematicians; Gutiérrez, 2012a; 2012b).

One relevant effect of conceptualizing mathematics in these ways is to disconnect

mathematics inquiry from the inequities both students and teachers face in and out of school. With these discourses of mathematics as abstract and universal influencing teachers, it is perhaps not surprising that most mathematics education in U.S. schools is characterized by teachers teaching and students following rote procedures that lead to a single and previously known (to the teacher) answer (Gutiérrez, 2012a). In these traditionally taught classes there is often little room for creative uses of mathematics or for connections between subject areas (Boaler & Greeno, 2000). Skovsmose and Valero (2001) describe mathematics teachers in traditional mathematics classes as autocratic and students as passive. The lack of creativity and connection can be alienating to any student who does not identify as "mathematical." Mathematics education has been specifically linked to the technological advancement of nations, and has remained so for over 50 years (Skovsmose & Valero, 2001). All student experiences, especially experiences that might be seen as culturally specific, are largely irrelevant to the class since the mathematics taught lacks contextual connection to student lives. In these classes students learn to see mathematics as unitary and certain, with minimal social dimension and minimal need to collaborate. Learning takes place on a very shallow level (i.e., only learning rote procedures and formulas) and students view themselves as passive learners. Some students go so far as to view thinking as only minimally necessary in mathematics, other than selecting the correct formula (Boaler & Greeno, 2000).

In response to the disconnect experienced by many students in traditional mathematics classes, there have been significant reform attempts in an effort to incorporate meaningful problems and contexts and to develop multiple understandings of mathematics (Ellis & Berry III, 2005; McClintock, O'Brien, & Jiang, 2005). Other

reform efforts have tried to increase the cultural sensitivity of teachers and increase their willingness to use classroom practices that are equitable for female, racial minority, and/or language minority students (Ellis & Berry, 2005). Despite their significant contribution to an understanding of mathematics education, these efforts have met with limited success and significant resistance among mathematics educators (de Freitas, 2008; Gutstein, 2006).

De Freitas (2008) has suggested that mathematics teachers may resist these reforms out of an interest to preserve the kind of mathematics that they themselves were comfortable with. In other words, they work to preserve the discourses of mathematics where knowledge is certain (Ernest, 1991) and static (de Freitas, 2013), in large part because they were taught in these ways and are comfortable and secure using these same discourses. Further, as these discourses also preserve and maintain White privilege there is inherent self-interest for the majority of mathematics teachers. However, Bartell (2013) notes that even mathematics teachers who are committed to teaching for social justice struggle to do so, because they end up focusing more on the mathematics despite their plans and intentions to do otherwise or, at best, they divide mathematics and social justice into separate parts of a lesson. Teachers, who want to teach in socially just ways, will be the focus of this study. It is possible that for these teachers the discourses of Whiteness and mathematics appear to be working against their desires to teach for social justice. Further these teachers may act in these ways without recognizing the contradictions of their behavior and desires. Yoon (2012) explains that Whiteness and the need to maintain White privilege can lead to behavior (in teachers) that is paradoxical and contradictory. In the case of mathematics teachers who plan to teach for social justice, but in the moment

are unable to, this may be the case.

Whiteness, Mathematics, and Discourses

Whiteness Theory operates on the assumption that the lives of all people in the US in particular (but elsewhere as well) are racially structured, including the lives of White[1] people (Frankenberg, 1993; Frye, 1992). Since mathematical achievement plays out on clearly racial lines in U.S. K-12 schools (Stinson, 2004) I operate from the assumption that race (as a social construction) is a significant factor in determining mathematical success as traditionally measured within mathematics (e.g., correctness, following procedures, grades, standardized test scores). Whiteness is a system of privileges, power, and authority that primarily benefits White people. The ways in which these privileges are built into the institutions and thinking of society has created a power structure that favors and is maintained through discourses. Discourses are these ways of thinking and being in society (Gee, 2005; 2012). The discourses of Whiteness are the "common sense," unquestioned (by most Whites, at least) beliefs and values about the way the world does and should work (Yoon, 2012). These discourses of Whiteness currently serve to deny the existence and relevance of racism and, as a consequence, preserve racist structures in the United States, as well as within schools and mathematics education.

Just as discourses maintain Whiteness, there are also discourses of mathematics and mathematics education that describe and exemplify the ways in which mathematics

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[1] Here, and elsewhere, in this document I will use White (capital W) to draw attention to White as a racial category. One of the ways that Whiteness operates is by directing attention away from Whites by suggesting that only people of color have something called "race."

should be taught and the ways in which mathematics should be learned. These discourses shape teacher and student understandings of what mathematics is. For example, if you are asked to think about mathematics you probably think of a school classroom, textbook, or equations on a paper or on a board. Each of these contexts (classrooms, textbooks, and equations) is dominated by common understandings of mathematics as abstract, apolitical, neutral, and acultural (Ernest, 1991; Walkerdine, 1988). In this context doing mathematics means manipulating symbols to arrive at a correct answer that has little to no meaning (Brown, 2001; Gutiérrez, 2012a).

Even so-called "application" problems in school mathematics typically have little connection to contexts that are meaningful to students or teachers. Because of the disconnect between mathematics and their experiences students are often unable to make meaningful connections between the mathematics they are learning in school and their struggles and interests outside of school. Even students who "correctly" answer these questions often do so by ignoring real-world considerations that make their mathematically correct answer irrelevant outside of math class (Mukhopadhyay & Greer, 2001). More importantly for students of color these understandings of mathematics are based on the White, male, middle-class perspectives (Ellis & Berry III, 2005: Walkerdine, 1988; 1990), which work to deny the relevance of perspectives of communities of color (Thompson, 1998). Thus beyond irrelevance for students of color mathematics is also potentially alienating. These discourses of mathematics as abstract, apolitical, neutral, and acultural interconnect and overlap (these connections will be explored in more detail later) with discourses of Whiteness that deny the existence of racism and view the world through a White lens. The ways in which these discourses

work together exclude both the possibility and the necessity of teaching mathematics for social justice.

Poststructural Perspectives on the Operation of Power

From a poststructural perspective power is multidimensional and productive. It resists simple definition as essentially two-dimensional (the power of the elites to control vs. the power of the people to resist). In this perspective one power structure is merely replaced by another, which then becomes dominant (Wang, 2011). From a poststructural perspective power is diffuse and exists throughout society in interactions between people. Thus, the ultimate goal in teaching mathematics for social justice is not to simply replace dominant perspectives on race or on mathematics, but instead to create a new system that works in fundamentally different ways towards "a positive relationship between mathematics, people, and equity throughout areas of the globe" (Gutiérrez, 2002a, p. 148). This goal requires a different understanding of the ways in which power operates. Power as productive means that it produces, but rather than producing things, power produces events and structures relationships between people. As teachers learn to think critically about their own discourses and exercise of power they may use their power in different, more socially just ways.

Teachers and students in a mathematics classroom together produce a particular event of mathematics education. This event (a moment of mathematics education) does not exist outside of the moment and efforts of these people; it is not an object. However, it also does not exist free of the influence of the context in which it occurs, which includes prior events of mathematics education (as engaged through discourses), understandings of what it means to be a teacher or a student, and understandings of race

and what it means to be White, Black, or Latina/o, etc., and understandings of what it means to be male or female. In this sense then each moment in a mathematics class is a recreating (as opposed to creating) from prior, historical moments in mathematics classes, but never the same as prior or future events and never completely formed (Wang, 2011). The discourses that teachers and students engage in this process influence the thoughts and actions of teachers and students in part through the beliefs and values embedded within them. Problems arise because when a teacher is familiar and comfortable with these discourses the underlying beliefs and values are mostly invisible (Fairclough, 2001). In the case of the mathematics teacher trying to teach for social justice the common-sense discourses of what it means to teach mathematics may appear to conflict with what it means to teach for social justice.

Why Focus on Mathematics?

There are multiple reasons to study the teaching of mathematics for social justice. Mathematics is a primary means of maintaining White privilege in schools through its gatekeeping role in determining who has access to high status fields (science, medicine, engineering, mathematics, physics, etc.) and higher education in general (Stinson, 2004). The maintenance of these privileges reinforces existing power structures with Whites in positions of power and with increased opportunities for wealth accumulation for White people. Further, mathematics may prove to be a powerful tool in the promotion of social justice (Gutstein 2006) by critiquing current inequities and exploring other ways of understanding and viewing the world. Mathematics as a subject area has been particularly resistant to efforts to promote social justice (de Freitas, 2008; Gutstein, 2006). While some subject areas have appeared to be more open to teaching for social justice,

mathematics, especially secondary mathematics, has not. The reasons for this resistance are only partly understood. A better understanding of the ways in which discourses of mathematics and discourses of Whiteness promote resistance to teaching for social justice is the primary aim of this study.

Research Questions

Traditional school mathematics is a problematic subject, particularly for historically marginalized students. School mathematics has been used to sort students into higher and lower tracks and to bar entry into higher education (Stinson, 2004). This sorting mechanism is accomplished through the discursive link between mathematics ability and intelligence. Through this connection mathematics achievement is used to create an intelligence hierarchy among students. Further, even those students who have been successful in school mathematics may have had to change meaningful aspects of their identities (Boaler & Greeno, 2000; Gutiérrez, 2012b) in order to advance. While social justice approaches to mathematics education have seen some success they remain only a small part of U.S. secondary mathematics education (Gutstein, 2006). The small impact of social justice mathematics on mathematics education is in part because of the consistent resistance to social justice education among mathematics teachers. But this resistance is not well understood (de Freitas, 2008) and may be due, in part, to the influence of the discourses of mathematics and Whiteness. As a consequence in this study I seek to understand how teachers' desires for equitable classroom practices are alternately facilitated and impeded by how they position themselves to teach mathematics for social justice and how they are influenced by the discourses of school mathematics in ways that work against their ideals of equitable practice. To this end this study seeks to

answer the following questions:

1) What discourses do secondary mathematics teacher candidates invoke when discussing social justice in their own teaching practice?

2) How do the discourses secondary mathematics teacher candidates use around school mathematics in the United States interact with the discourses they use around social justice mathematics?

3) How do secondary mathematics teacher candidates merge/manage and challenge the disparate discourses of mathematics and social justice during student teaching in a program that emphasizes preparation for teaching in culturally and linguistically diverse contexts?

CHAPTER 2

LITERATURE REVIEW

In this chapter I review some of the relevant research on social justice mathematics. I first explain the principal aims of social justice mathematics and the challenges of teaching mathematics for social justice. I then explain how the discourses of dominant mathematics contribute to the challenges of teaching mathematics for social justice. I demonstrate how the dominant discourses of the nature of mathematics are inconsistent with an historical understanding of the development of mathematics. This historical piece further demonstrates the potential political and practical nature of mathematics, and thus that mathematics does not have to be incompatible with social justice education.

Teaching Mathematics for Social Justice

Gutiérrez (2002a) uses the term "dominant mathematics" to describe the mathematics that is traditionally taught in schools, that serves dominant interests in maintaining the status quo of society and schooling, takes an uncritical approach to the structure of society and schooling, and fails to recognize the contributions and potential contributions of marginalized peoples. Drawing from her definition I will use two terms throughout this study. First, I use "school mathematics" to refer specifically to the mathematics that is taught in schools. School mathematics is part of, but not all of,

dominant mathematics. Second, I will use "dominant mathematics" to refer not only to the mathematics that is taught in schools, but also to mathematics outside of schools, as it is used by professional mathematicians as well as other professionals whose work depends on mathematics (engineers, quantitative scientists, actuaries, etc.).

The need to improve the mathematics educational outcomes of historically marginalized groups of students is largely unquestioned, with various mainstream organizations calling for improvement in mathematics education for decades (Gutiérrez, 2002b; Secada, 1989). However, social justice mathematics advocates argue that social justice education must go beyond merely improved educational outcomes (Ebby, Lim, Reinke, Remillard, Magee, Hoe, & Cyrus, 2011; Frankenstein, 1990; Gutiérrez, 2002a; 2009; 2012a; 2013; Gutstein, 2003; 2006; 2007; 2009; Moses & Cobb, 2001; Secada, 1989; among others). Gutstein (2006) explains that even when students of color and other marginalized groups have been successful in mathematics classes, they have not learned to use mathematics to challenge the inequalities that directly affect them. The result is that the status quo of inequality is maintained for the majority of marginalized students. From this perspective a social justice approach to teaching mathematics is a necessary part of achieving more equitable outcomes in mathematics education and, more importantly, for the creation of a more just society. The work of Frankenstein (1990) was ground breaking in linking critical pedagogy and ethnomathematics to U.S. mathematics education, through what she calls *critical mathematical literacy.* Frankenstein's work became the foundation for what I refer to here and throughout the text as social justice mathematics. The examples of social justice mathematics as lessons that engage in social (both local and global) critique as shown in Gutstein's work (2006; 2013) and

Frankenstein's (1990) have become the most well-known approach to social justice mathematics.

However, there are others who promote a broader view of social justice mathematics. These include Garii and Appova (2013) who distinguish between teaching in a socially just manner (equitable access through pedagogy), teaching about social justice (social critique), and teaching for social justice (making connections to students lives). Skovsmose and Valero (2001) use the term "democratic" mathematics education to discuss similar ideas to those of social justice mathematics. They define democracy in terms of collectivity (collective action), transformation (collective work for change), deliberation (as dynamic dialogue), and cofleciton (collective reflection). Skovsmose and Valero link these ideas to mathematics education by suggesting that we rethink what mathematics are needed for democratic citizenship, teacher-students relationship in mathematics classes, what mathematics education means in a school, the national role of mathematics education, and the global role of mathematics education.

I draw primarily from the work of Gutiérrez (2002a; 2002b; 2009; 2012a; 2012b; 2012c; 2013; 2015) and Gutstein (2003; 2006; 2007; 2008; 2009; 2012). Gutstein's work is important, in part, because of the concrete examples of teaching mathematics for social justice that he gives and because it is perhaps the best known form of social justice mathematics in U.S. mathematics education. The work of Gutiérrez encompasses a variety of approaches that are relevant to the U.S. context and focuses on the school-teacher-student level that is most applicable to this study. A social justice approach to mathematics education has demonstrated the potential to addresses some of the current issues in mathematics education (Gutstein, 2006). Social justice approaches to

mathematics have been developed to address societal inequities and to increase access to advanced mathematics for women, students of color, and poor students. Gutiérrez (2002a) defines social justice mathematics as working towards a goal of:

> Coordinat[ing] (a) efforts to get marginalized students to master dominant mathematics with (b) efforts to develop a critical perspective among all students about knowledge and society in ways that ultimately address (c) a positive relationship between mathematics, people, and equity throughout areas of the globe. (p. 148)

The first portion of this goal is in reference to the wide disparities in achievement in mathematics classes that have disproportionately affected students of color and students living in poverty. This part of her goal then is to achieve equitable outcomes (as evidenced by traditional measures of achievement) in mathematics courses. Many mainstream organizations such as the National Council for Teachers of Mathematics (NCTM) also push this goal (Secada, 1989). However, the second portion of this goal goes beyond outcomes to include teaching students how to use mathematics as a tool for social critique. Historically mathematics has been used to justify inequalities. Here Gutiérrez argues that students should be taught to use mathematics to argue against inequalities and critique the mathematics used in arguments that maintain or exacerbate current inequalities. The third portion is a more distant goal and, by her own admission, is a kind of mathematical utopia in which mathematics has been used to achieve full equality across the globe.

Gutiérrez (2002a) notes that Eric Gutstein has developed a curriculum that perhaps best achieves the first two goals. Gutstein (2006) explains that his work in Chicago with low SES students of color has focused on working towards a more equitable society, achieved through greater academic success and access to advanced

mathematics and higher education for students of color. However, his goal goes beyond success and access to teach students to use mathematics to argue for a more just society as well as critique arguments for maintaining the status quo. To accomplish these goals he has developed curricular materials that make mathematics relevant to issues that the students face and connect them to more global issues. These local issues provide the rich, meaningful details that are lost in abstract school mathematics. He then works with students to use mathematics to understand and argue against the inequities that they face, such as poverty, racial profiling, and gentrification of their neighborhoods.

In his own work Gutstein (2006) draws on the Freirian tradition of education for liberation and defines social justice mathematics as working towards both mathematical and social justice goals. The social justice goals can be summarized as saying that students should be able to use mathematics to better understand their world (especially inequities), construct mathematical arguments against those inequities, and counteract the effects of deficit racial perspectives in society. The mathematical goals can be summarized as saying that students should develop positive attitudes about mathematics and understand college-preparatory mathematics to achieve traditional academic success. Here Gutiérrez (2002a; 2012b) might suggest an additional goal of "writing the mathematical word" to acknowledge that students of color can also shape the field of mathematics and make contributions to mathematics. Otherwise their goals overlap in many aspects.

Difficulties of Teaching Mathematics for Social Justice

While Gutstein (2003; 2006; 2007; 2012) in particular has demonstrated impressive successes through social justice mathematics curricula, his work, as well as

the work of other social justice mathematics educators (Ebby et al, 2011; Gutiérrez, 2002a; Moses & Cobb, 2001), has received relatively little attention from the mainstream segment of mathematics education research (Gutiérrez, 2002a). Mathematics teaching as a whole has not seen widespread acceptance of mainstream reforms (McClintock, O'Brien, & Jiang, 2005) much less of social justice education (Gutstein, 2006).

Both Gutstein (2006) and Gutiérrez (2009) explore the difficulties of teaching for social justice within a mathematics context. However, they do so in different ways. Gutstein (2006) lays out what he views as the characteristics and knowledge that a mathematics teacher should exhibit in order to effectively teach for social justice. He argues that mathematics teachers will need to go "beyond the mathematics" in order to build understanding of social and political forces that structure a seemingly straightforward or neutral situation. For example, in one project his students investigated the potential effect on home prices of a proposed gentrification project in their neighborhood. To go beyond the mathematics in this situation requires a knowledge of some of the local area history, the way home prices and mortgages work, and the history of racially segregated neighborhoods in the US. Both teacher and students then need political knowledge, historical knowledge, and economic knowledge in addition to the mathematics that are necessary to understand just the numbers of gentrification. Not only is this knowledge not part of a traditional teacher preparation program, but because it is perceived as nonmathematical knowledge mathematics teachers may see it as irrelevant to their teaching, or even inappropriate.

Gutstein (2006; 2008) further advocates that teachers develop "political relationships" with their students. This includes taking a political stance in support of

students, but also requires making that political stance known. This may be difficult for teachers who view mathematics as inherently apolitical and particularly so if they value their relationship with mathematics over their relationship with students (Gutiérrez, 2009). But even when teachers have the necessary knowledge and work to develop political relationships with students they may have students who resist their efforts, because students also may have learned to view mathematics as apolitical and decontextualized, through their previous experiences with school mathematics. A final difficulty that both Gutiérrez and Gutstein (2006) explain is that, in the current education system, teachers must still find ways to prepare students to take and pass high stakes tests as well as prepare them for success in future mathematics classes. Without this preparation students' educational opportunities will grow more limited. This requires some kind of negotiation of the discourses of school mathematics and teaching for social justice. These negotiations may be facilitated as teachers begin to challenge the ways in which mathematics may push them away from teaching for social justice.

Despite the scope of his work mathematics teachers tend to associate the work of Gutstein (2006) only with the social critique lessons that he presents. The work of Gutiérrez (2009) is useful in creating a more complete understanding of what it means to teach for social justice. As she explains, teachers must recognize the importance of knowing their students well, including their culture, history, and personal background, in order to incorporate and validate those experiences in the classroom. Simultaneously, teachers must also recognize that they can never truly know their students because their students are continually changing, not static individuals or essentialized representatives of a culture. This stance complicates the discourse of teacher as knower. Second, teachers

must take charge of the curriculum in the classroom, how it is presented to the students, and use all of their skills to make the mathematics engaging and inviting to the students, but they must recognize that it is ultimately the students' decision to participate or not. This tension recognizes that both students and the teacher have power in the classroom. Too often a textbook, department culture, school policies, district policies, or state policies dictate what a curriculum will be and how it should be taught, resulting in a disengaged teacher and/or disengaged students. Finally, Gutiérrez notes, as does Gutstein (2006), that there is a tension between teaching students (i.e., meeting their needs and interests in addition to seeking social justice) and teaching the necessary mathematics to meet standards or to prepare students for the next level of mathematics and college. This last point is problematic because not preparing students may limit their education, but the standards do not recognize the value of teaching for social justice. These tensions are not meant to be resolved; instead Gutiérrez (2009) advocates that teachers hold on to both sides of each in order to teach for social justice.

These difficulties highlight the possibility that there is something about mathematics that influences teachers in ways that make teaching mathematics for social justice difficult. Various authors who write about the nature and philosophy of mathematics and mathematics education provide perspectives on what it is about mathematics that make teaching for social justice particularly difficult in this subject. Rousseau and Tate (2003) make one of the more direct links noting that the philosophy and foundation of mathematics discourage reflection on issues of social justice because they normalize those same issues. For example, by presenting mathematics as neutral (not biased) the consistent achievement gap between student groups appears to be natural.

However, this does little to explain what it is about mathematics that makes the normalization of social injustice possible (after all this happens in other subjects as well). Brown (2001) explains that mathematicians, like other groups, form a kind of community with mathematics as their language. Since the language of mathematics is created and re-created by mathematicians, the beliefs and values of those mathematicians are part of the discourses of dominant mathematics. In this case then dominant mathematics was created with the beliefs and values of and in the image of a select group of wealthy, White, males and is reinforced through abstraction, decontextualization, and the use of a formal, symbolic language (Walkerdine, 1990). Historically these wealthy, White, and (often) state-sponsored mathematicians have policed what was recognized as mathematics and viewed other approaches to mathematics as a threat to the bounded and absolute mathematics that they had created (de Freitas, 2013). This defensive behavior protected their own positions by excluding other possibilities and in the process they shaped mathematics as the absolute and apparently apolitical mathematics taught today, by closing off other possibilities. It is from this absolute nature of dominant mathematics that potential conflicts with teaching for social justice may arise.

Understanding Dominant Mathematics

Ernest (1991) claims that mathematics is constructed as the most certain of human knowledge. The dominant view of mathematics for millennia has been an *absolutist* view. Ernest defines this absolutist view as maintaining that mathematics is made up of certain knowledge and absolute truth. Further because these truths can be established without reference to empirical evidence they are the most certain of any knowledge. The absolutist view of mathematics has dominated mathematics education and is present in

most textbooks and classrooms in the United States. Brown (2001) specifies that in this absolutist world mathematical terms do not refer to anything tangible. For example, the symbol "5" represents a numerical value. While people may use the symbol to refer to the quantity five of something, the symbol on its own does not refer to anything concrete. This lack of referent is part of the certainty and abstractness created in modern mathematics. Further Brown (2001) points out that mathematical objects are imaginary. This is perhaps most clear in Euclidean geometry, which is literally an imagined world in which points have no dimensions and lines have no thickness. Everything is an idealized form that cannot exist in a tangible form. It is possible that the decontextualization and abstractness used to create these idealized forms make it difficult for teachers, students, and others to connect mathematics to the highly contextualized reality of social inequality that is required by social justice education.

Dominant Mathematics Complicates Social Justice Efforts

Mathematics education may be particularly resistant to social justice approaches, because mathematics teachers are dependent on the discourses of abstract certainty that arise from a discourse of mastery and certainty that is only possible within abstract mathematics (de Freitas, 2008). The work of de Freitas (2008) suggests that, in the case of mathematics teachers, resistance to social justice approaches to education may stem, in part, from the teachers' connections to and investments in mathematical discourses. In the case of mathematics this resistance may be facilitated by the dominant discourses around school mathematics that can cause conflict between a desire to teach for social justice and what they feel falls within the discourses of dominant mathematics. This could push mathematics teachers to resist explicitly political ideas, such as social justice, because

they pose a perceived threat to their *mathiness*—the mathematical part of their identity. Mathematics teachers may feel a particular need to defend their *mathiness* because they are juxtaposing a high-status field (mathematics) with a relatively low-status field (teaching; R. Gutiérrez, personal communication, May 13, 2013).

A Perception of Mathematics as Apolitical

The potential conflict between school mathematics and social justice may lie in the perceived apolitical nature of school mathematics. This apolitical characteristic of mathematics depends on a view of mathematics as neutral, abstract, certain, and absolute. Walkerdine (1990) refers to this understanding of mathematics as a kind of axiomatic, rule-bound world in which mathematical objects and forms have consistent definitions which can be depended on and proved. These conditions are necessary for the certainty with which mathematics operates. Despite the real effects for many students, this mathematical world is a fictional world, because of its lack of connection to anything tangible. To enter into this fictional world a problem needs to be stripped of the context in which it arose including the needs and perceptions of the people for whom the problem is meaningful (Walkerdine, 1988; 1998).

In this way, a mathematical problem that addresses the lack of resources common in many urban schools is unlikely to be recognized as valid within the discourses of dominant mathematics because the numerical comparisons between schools are not mathematically interesting; instead they are *politically* meaningful. However, problems that are often challenging for students such as the now legendary "If a train leaves point A at a certain speed and another train leaves point B at the same time at a certain speed when will they meet?" is likely to continue as a regular part of the mathematics

curriculum because of its perceived mathematical value and despite its lack of relevance or meaning in the lives of students. When contextualized and meaningful problems are allowed in they are always on the periphery. It is only after meaningful problems have been generalized and abstracted that they become mathematically valuable (Walkerdine, 1998). However, this generalization and abstraction reinforce the view of mathematics as universal and apolitical, thereby hiding the way the dominance of these discourses excludes social justice.

Problems Arising From an Apolitical View of Mathematics

The dominant view of mathematics as absolute and certain grows out of an ancient Greek mathematical system that valued formal, abstract mathematics over practical, everyday mathematics (Cooke, 2013; Seife, 2000). The result today is that practical or applied mathematics is often not recognized as fully in the mathematical world despite the complexity of practical mathematics. Practical mathematics are influenced by so many variables that situations can never be completely predictable or certain. They are in this way a challenge to the certainty of absolute mathematics. Often those who enjoy school mathematics, enjoy this certainty (Boaler & Greeno, 2000). However, a common complaint about school mathematics is the refrain, "when will I ever use this?" The problem is that applying school mathematics to real contextualized problems is complex and does not usually look like the mathematics that is found in textbooks and taught in schools. As a consequence contexts that may heavily involve mathematics are not recognized as mathematical. Rose (2012), for example, describes the complex mathematics found in a vocational welding program,

> The central precepts of welding are travel—the speed of your movement of the

> instrument—the distance of the instrument from the metal, the angle of it, and how hot you've got it. . . . Travel, angle, and all that are further complicated in some processes by the fact that the electrode conducting the current is being used up as you weld, so you've got to continually adjust your travel speed and angle and distance to keep things constant. (p. 9)

Speed and distance are both directly related to algebra, while angle is a key concept in both geometry and trigonometry. Burn rates and temperature also involve complex mathematics. Yet what these welders do and what they are taught is not recognized as mathematics, sometimes even by those who do it. Rose describes the instructor of this welding course as saying that

> he doesn't know mathematics very well. The ideal, he believes, would be to have a mathematics teacher demonstrating the division of decimal fractions and calculation of volume, and explaining the *why* of what the class is doing, the mathematical principles involved. (p. 12)

In a related way, Frankenstein and Powell (2002) explain that the problem of knitting the crook of a sock without the material bunching up is mathematically similar to the problem of creating a curved pipe without the metal folding in on itself. While the problem of a curved pipe associated with "man's work" is engineering, the knitting, seen as "women's work," is not even mathematical. So why is it that the instructor does not consider what he does to be mathematics? Why is it that knitting a sock is not considered mathematical? It is likely that neither the welders nor the knitters are making mathematical calculations in their heads, nor are they solving equations; without these formal markers of abstract mathematics most of us are unable to recognize knitting or welding as mathematics. However, good knitters and welders take all of these factors into account and make adjustments as they go. Because both the mathematics of welding and of knitting socks are not recognized as mathematics, welders and knitters may be left with the idea that they are not mathematical and that they are not as intelligent (because

we equate mathematical ability with intelligence) as mathematicians. Clearly these decisions of what counts and what does not count as mathematical are political and they have political effects.

As in the previous example from Rose (2012), mathematics teachers may be unaware of how an absolutist view of mathematics obscures their ability to recognize mathematics without its formal markers. In fact these teachers may also be caught up by discourses of abstract mathematics just as students are (Walkerdine 1998; Walshaw, 2013). Further, mathematics teachers may feel a need to preserve these discourses. Teachers of mathematics may enjoy the distinction that mathematics gives them over other teachers and over their students. They can feel a sense of control and mastery from their ability to solve mathematical problems and manipulate mathematical symbols (de Freitas, 2008; Walkerdine, 1998). Changing that (already political) world by introducing politics explicitly, to take a social justice approach, or students' experiences and perspectives, to take a culturally relevant approach, can threaten the control that teachers gain from formal mathematics and the system of White privilege that formal mathematics helps maintain. Teachers and Whiteness also benefit from the appearance of neutrality inherent in absolutist mathematics that allows them to hold themselves above the political opinions of others and other subjects. Making mathematics explicitly political can threaten this neutrality and mathematics would lose the illusion of colorblindness.

By using mathematics to maintain differences between themselves and their students and between themselves and other teachers, mathematics teachers create a kind of authority, both literal and moral. The literal authority accompanies the traditional positioning of teachers. The moral authority derives from the perceived neutrality of

abstract mathematics. This neutrality (lack of bias) allows mathematics teachers to perceive and present themselves as fair judges in the classroom. This role of unbiased judge is one of the privileges that accompany and maintain Whiteness (Frye, 1992), because it allows White people to judge wrong and right, including what acceptable responses are in a mathematics class. The authority to judge right and wrong closely overlaps with discourses of dominant mathematics that portray mathematics as absolute and certain. The mathematics teacher, as representative of this certain world, is required by the discourses to judge students as right or wrong mathematically. This privilege of judgment then determines which students advance academically and shapes how students perceive themselves in relation to mathematics. However, this authority is created by stripping away the human contexts in which mathematics operates and from which it arises (Cooke, 2013; Walkerdine, 1988; 1998) and it is only within a fictional world that mathematical authority can exist; as such it is always precarious and must be reasserted.

When this authority is threatened, those who have mastered the abstract discourse of school mathematics can retreat into the formal, symbolic language to reassert their supposed superiority. In the mathematics classroom a teacher can, without realizing it, use this power to keep students in their place and maintain their own authority. Teachers can use this power to justify and normalize the failure of students—especially minority and low-income students (Martin, 2007; Rousseau & Tate, 2003)—and in the process maintain a sense of fairness and justice based on the neutrality of mathematics. In this manner, teachers may use the authority of school mathematics to justify and rationalize the injustices perpetuated by inequitable practices in schools. Mathematics aids in this normalization process so that mathematics teachers may participate in failing students

along racial lines, while maintaining a self-perception as just and fair, because their judgment rests on the neutrality of mathematical discourse. Recognizing the role of mathematics and the role of teachers of mathematics in creating these injustices threatens the comfort of moral authority that teachers may gain from the fictional world of school mathematics. For these reasons, among others, the formal, symbolic language of mathematics continues to be privileged and the White, middle-class, male discourses of mathematics continue (Walkerdine, 1988; 1998).

While skilled use of mathematical discourses grants access to power and authority within the real world, that access serves to divide students as mathematical or not (Skovsmose & Valero, 2001). These divisions create an over-representation of students of color and women as "not" mathematical. It is exactly the abstract within mathematics that is pointed to as the pinnacle of thinking that those perceived as not mathematical (whether White students or students of color) are unable to obtain (Walkerdine, 1990). Within this world alternative ways of thinking and solving problems are not viewed as mathematical, are looked down on, and ultimately discouraged (Gutiérrez, 2012b). Gutiérrez notes that often if a student in a middle- or high-school geometry course suggests that not all lines are straight (a perfectly logical conclusion from the real-world) that understanding is often quickly corrected to fit the constraints of classical, Euclidean geometry. However, mathematicians have used the simple understanding that not all lines are straight to explore other geometries (hyperbolic, spherical, taxi-cab, etc.), the existence of which is unlikely to be mentioned in secondary geometry courses.

The result too often can be that students may stop thinking of alternatives, focus on the information given, and come to believe that their understandings and experiences

are not valid within the mathematical world (Boaler & Greeno, 2000). Students thus must "become someone else" (Gutiérrez, 2012b, p. 30) to succeed. Students are actively discouraged from using the kind of thinking that views knowledge as multiple and making connections in the ways that mathematicians do (Boaler & Greeno, 2000; Gutiérrez, 2012a; 2012b). These students may feel that to succeed in mathematics they must think and reason in ways that are more like an elite White male view. Just as mathematics, historically, was held back by an unwillingness to accept the concept of zero (Seife, 2000), Gutiérrez (2002a; 2012b) has argued that modern mathematics is held back by an unwillingness to recognize the potential contributions of women and students of color. While abstract mathematics need not be the only kind of mathematics, it continues to function as if it were. As a consequence, school mathematics has remained relatively unchanged despite decades of reform efforts (Gill & Boote, 2012). Both teachers' and students' thinking is constrained by the maintenance of the discourses of absolutist mathematics. However, this view of mathematics is inconsistent with an historical understanding of the development of mathematics.

An Historical Perspective of Mathematics as Political and Practical

A review of some of the history and development of mathematics helps to dispel some of the myths about mathematics that are perpetuated by the dominant discourses. In general, these discourses portray mathematics as apolitical, abstract, certain, and neutral. It is the most decontextualized of the sciences and gains its prestige from this abstraction and decontextualization (Gutiérrez, 2012b; Walkerdine, 1988). However, mathematics has not always been neutral, apolitical, and abstract. Cooke (2013) explains that

mathematics is created anytime people think about their world. In particular he credits accounting, surveying, astronomy, and kingdoms (including laws and theology) with creating the roots of mathematics. Joseph also links the development of Indian mathematics (1997) and Mayan mathematics (2008) to religion and astronomy. He further connects the development of aspects of Chinese mathematics to the surveying of land (1997). All of these practices are political, contextual, or practical if not all three. An extreme example may illustrate the political nature of mathematics:

> Hippasus of Metapontum stood on the deck, preparing to die. Around him stood the members of a cult, a secret brotherhood that he had betrayed. Hippasus had revealed a secret that was deadly to the Greek way of thinking, a secret that threatened to undermine the entire philosophy that the brotherhood had struggled to build. For revealing that secret, the great Pythagoras himself sentenced Hippasus[2] to death by drowning. To protect their number-philosophy, the cult would kill. (Seife, 2000, p. 26; see also Joseph, 1997)

That secret was the existence of irrational numbers[3] and their existence went against Greek understandings of how the world worked. For the ancient Greeks number and shape were directly linked and formed a kind of religion. Those who followed it gained power and prestige, which was resented by others, eventually resulting in Pythagoras' death (Seife, 2000). Dominant mathematics, especially as represented in schools, has its roots primarily in this Greek system. This Greek influence is seen most clearly in the lasting impact of Euclid's *Elements,* which lays out his approach to the study of geometry (Cooke, 2013). The *Elements* is, in a literal sense, a fictional world in which a point has no dimension and lines continue on straight, and infinite. The world described in the

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[2] Whether he was actually killed is disputed; he may have been only banished from the society (Veljan, 2000).

[3] Irrational numbers are decimal numbers that do not repeat and do not end. Some of the more commonly known irrational numbers are π, e, and $\sqrt{2}$. It was $\sqrt{2}$ that was particularly troubling to Greek geometers.

Elements bears only a passing resemblance to the world we live in, because that was not its purpose. According to Cooke (2013) what set Greek mathematics apart was its formalization, which is only possible through abstraction and decontextualization.

The formal development of mathematics (as opposed to a practical or applied or musical, etc. development) has a continuing influence on the development of mathematics and has set the standard (in the Western world) of how mathematical knowledge is to be judged. Greek mathematics was clearly political and probably religious. The Greek system is probably best recognized currently in the form of the Pythagoreans, who viewed shapes as having characteristics of good and evil (Cooke, 2013). This understanding of mathematics is the foundation of modern school mathematics. In this system the abstract was venerated, while the practical was denigrated. It was this mathematical belief system specifically that pushed Western mathematical philosophy to reject zero, resulting in centuries of set-backs in mathematics, science, and economics. While zero was used to advance Indian and Mesoamerican mathematics, mathematics in Europe was hampered by the lack of a place value system that zero makes possible, among other mathematical advantages of using zero (Joseph, 2008; Seife, 2000).

Political Efforts to Shape the Development of Mathematics

An unwillingness to accept zero led to experiences that were somewhat comical. Personal and societal beliefs about zero at times resulted in ridiculous behavior, such as Greek mathematicians who translated problems involving fractions into the Babylonian systems in order to surreptitiously use zero, but then translated problems back to the Greek system to hide their use of zero (Seife, 2000). But there have also been more

serious consequences both for society and individuals. Political differences over the use of zero have arisen periodically in the history of mathematics (Joseph, 2008). Mathematical history is filled with power struggles such as that between Newton and Leibniz over the invention of calculus. Since the English chose to follow Newton, with his more awkward notation, they quickly fell behind the rest of Europe in mathematical prowess. The problem was caused in part because Newton held on to his work for years after its development. He feared the political-religious ramifications of work that depended so much on zeros and infinities. After his work the church of the day complained about this use of zeros and denounced it as heretical. Newton (like other mathematicians) was unsure about the zeros in his work and he tried to hide them away in his calculations, resulting in the conflict with Leibniz (Seife, 2000), which may have contributed to the awkwardness of his notation. The conflict over the origin of calculus and who to follow were not isolated incidents. Power struggles were not uncommon and resulted in one mathematician being committed to a mental institution in one case while both church and state tried to ban the use of zero in another (Seife, 2000). In many ways political, religious, and personal values shaped the development of dominant mathematics.

Mathematics Arises From Practical Situations

While dominant mathematics, with its Greek roots, idealizes the abstract, it is often through practical problems, rich in contextual detail, that mathematical innovation has developed. For example, Italian workmen used pumps to raise water out of canals, but despite their efforts they could not get the water over 33 feet. Pascal was later able to determine that this had to do with the pressure of the atmosphere pushing down on the

water; the pumps were not able to overcome this pressure. Johannes Kepler developed methods for finding the volume of three-dimensional objects in order to more accurately calculate the volume of wine barrels. Egyptian mathematicians developed ways to calculate areas of triangles and rectangles in order to place property boundary markers after previous ones had been washed away by the flooding Nile River (Seife, 2000). While Seife recognizes Egyptian contributions to mathematics (especially as the forerunner of Greek mathematics), he also denigrates it, because it was practical and did not embrace abstraction, as the Greeks would. These same Egyptians were likely among the first (and certainly before Pythagoras) to use what is now known as the Pythagorean Theorem, but because their use was practical and Pythagoras abstracted and generalized it, he is the one who gets the credit (Veljan, 2000). The idea of an average arose from the need to divide the losses of merchant ships equally among the investors (Rubenstein & Schwartz, 2000). The innovation of the vanishing point in art, is essentially a zero, and allows the representation of three-dimensional space on a two-dimensional canvas (Seife, 2000). The vanishing point is just one of many uses of mathematics in art. However, because we do not see any formal calculation in the process of creating art it is not recognized as mathematical.

Conclusion

Social justice approaches to mathematics education have potential to improve the educational and life opportunities for students who are perceived as nonmathematical. This is particularly true for students of color and women. However, the dominant perceptions of mathematics exclude the possibility of mathematics that addresses social justice, even though these perceptions are inconsistent with the practical and political

development of mathematics. These perceptions lead to mathematics teachers who are resistant to mathematics education that is clearly political and contextualized. Social justice mathematics is potentially threatening to mathematics teachers' position in society, to the structures that maintain White privilege, and to dominant perceptions of mathematics. Thus mathematics teachers who want to teach for social justice face numerous obstacles in understanding what it means to teach for social justice. For teachers who have been schooled in and grown up with a static, absolute view of mathematics, as is standard in school mathematics, this shift in perspective will be difficult and will involve unlearning some of what they already know. I struggled (and still struggle) in my own teaching to work with my mainly Latino immigrant students as they tried to balance learning what they needed to advance academically with making mathematics relevant and critical.

CHAPTER 3

DISCOURSES OF WHITENESS IN MATHEMATICS TEACHER PREPARATION

Discourses

The concept of discourse is central to the questions that I am asking in this study. In focusing on discourse I draw first from the work of Gee (2005; 2012) and Fairclough (2001). Gee distinguishes between discourse (with a lowercase 'd,' to refer to individual speech acts)[4] and Discourse. Discourse (capital D) includes particular ways of speaking (such as, but not only, discipline specific speech), but also all the things that accompany speech that help us make meaning of the spoken word and of the speaker. These things include dress and appearance, the objects and tools that someone uses, the location or context of the interaction, and the ways in which people interact. Together these aspects of discourses allow someone to take on a particular role. In the case of a mathematics teacher then we recognize someone as a mathematics teacher not just through what she/he says but also by where the teacher is (in a classroom, in a school), the way the teacher is dressed (probably not formal, and not too casual, but somewhere in between), the objects the teacher has and uses (chalk, markers, red pen, worksheets, computer, etc.),

[4] I do not maintain this distinction preferring to use the uncapitalized term since each time I use the term I use it in the capital "D" sense.

the way the teacher interacts with other people in the school (his/her own students, other students, administrators, etc.), and the ways others interact with the mathematics teacher. All of these aspects of discourse hold for teachers generally, at least in U.S. public schools.

Mathematics teachers, however, have additional discourses to take up to get recognized as mathematics teachers and so to distinguish themselves from other kinds of teachers. They will be expected to use the symbols and language of mathematics. They will be expected to teach in specific ways, often ways that are not conducive to teaching for social justice. They probably will not be expected to discuss politics or bring political issues into the classroom (whereas a social studies teacher is, and other teachers might). Explicit politics are likely excluded by the discourses of mathematics education, in order to maintain a perception as apolitical. If this teacher brings social justice issues into the classroom she/he runs the risk of no longer being recognized as a mathematics teacher, potentially resulting in dismissal from the profession. It is important to note that most of what makes up discourses, like the discourses of high school mathematics teachers, is not explicitly taught. Instead someone who wishes to become a mathematics teacher must *continually* pick up these things through life experiences and observation, because the discourses are constantly shifting. This necessary work to be recognized as a mathematics teacher creates pressure and constraints on what a mathematics teacher can do. As Gee (2005) explains, "The key to Discourses is 'recognition' . . . . Whatever you have done must be similar enough to other performances to be recognizable" (p. 27). As you interact with others and receive (implicit) feedback that they either recognize your discourse or not; you must continually make adjustments in order to maintain the successful use of

that discourse. Each moment of discourse use is a recreation influenced by, but never exactly the same as past moments.

The need for similarity in successive reproductions of discourse use means that it is difficult to make significant changes in a discourse without your use of that discourse becoming unrecognizable (Fairclough, 2001). As a mathematics teacher if I deviate too radically from the discourses of mathematics teaching I risk being perceived as using other discourses altogether. However, if the changes are gradual enough (and include a large enough group), while engaging enough of the standard discourse then that discourse can begin to shift and change. As a consequence our ability to freely disrupt or change any discourse is constrained by what is recognizable to others—our shared understanding, which is influenced by how that discourse is historically understood. However, Gee (2005; 2012) does little to explicitly address the constraining power of discourses. Instead I turn to Fairclough (2001) to explain how power operates in and through discourse. While Fairclough does not use quite the same terms as Gee (2005; 2012) I will continue to use discourse as defined by Gee. As Fairclough (2001), drawing on Foucault, explains, the need to use discourses in a recognizable way forms a set of constraints that both restrict what is possible, but also enables action and interaction.

Discourses Enable and Constrain

The discourses of mathematics education both lay out the conditions of what it means to be a mathematics teacher and limit what a mathematics teacher can do and say and still be recognized as a mathematics teacher. In these ways discourses both enable and constrain (Fairclough, 2001). Discourses enable in the sense that taking up the appropriate discourses allows me to be seen and understood as a mathematics teacher.

However, they simultaneously constrain in that they set limits as to what speech, behavior, dress, actions, etc. are acceptable in being a mathematics teacher. This simultaneous enablement and constraint illustrate the operation of power through discourses. As we invoke various discourses we engage in a continual negotiation of power relations (Baxter, 2002).

In the mathematics classroom this negotiation occurs most directly between the teacher and students, but they also include other members of the school community and society wide perspectives. These negotiations are implicit and dependent on a shared idea between a teacher and students of what "teacher" and "student" means. This shared understanding is necessary for a discourse to enable interactions. However, if a teacher tries to politicize mathematics or teach in a way that validates various, nondominant perspectives, methods, and ways of thinking he/she runs the risk of not being recognized as a mathematics teacher by the students, by their parents, by other mathematics teachers, or by administration. These other participants (parents, other mathematics teachers, and administrators, etc.) are also part of this negotiation of power even when they are not physically present.

Consider this description of how discourses operate and how we use them. We are immersed in a sea of discourses through our continuous interactions with people, media, and institutions (see Figure 1). Some of them are dominant (capital "D") others are not dominant (lowercase d). Many of these dominant discourses are discourses of Whiteness since they maintain White privilege. The dominant discourses are connected via large block arrows (imagine varying sizes of arrows connecting discourses to the person) to show how through their ubiquity and regular, consistent repetition we

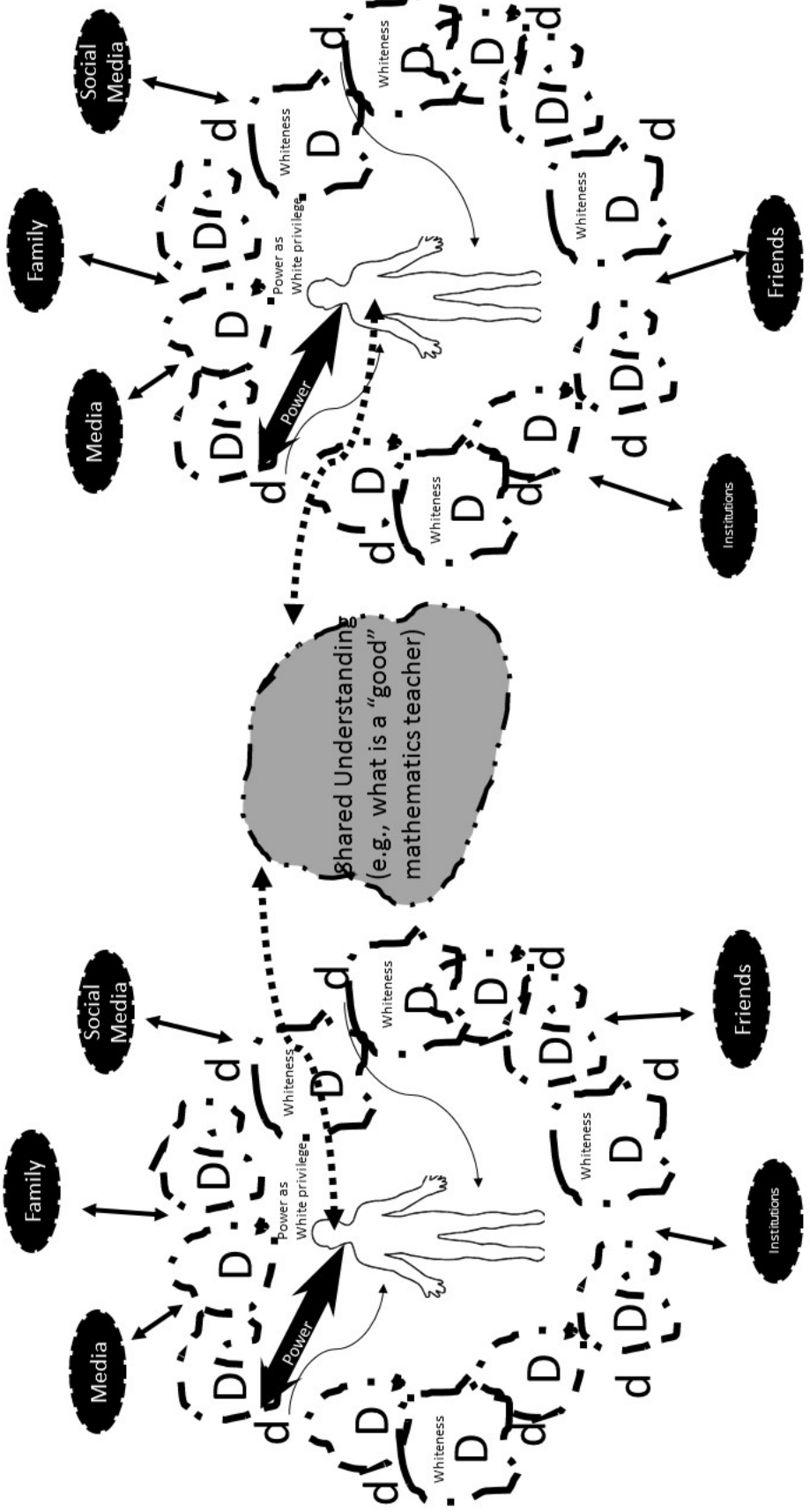

Figure 1 Discoures in Interaction

have much greater access to them. These discourses carry power with them that includes a sense of "goodness" (e.g., what it means to be a "good" mathematics teacher). The connections between us and the nondominant discourses are thinner and more tenuous. When we interact with someone we each draw on the discourses that we have access to and put them together to re-create in that moment our shared understanding of our topic of discussion (e.g., what it means to be a "good" mathematics teacher). To be able to discuss it meaningfully we need to have some shared discourses in order to understand one another (enabled), but this also means that we are limited (constrained) in our ability to imagine our topic of discussion in a way that differs from what is provided to us in the discourses that we know and have been exposed to.

Importantly through discourses we re-create significance, activities, identities, relationships, politics, connections, and sign systems and knowledge. Gee (2005) calls these building tasks. Further when we talk and interact with others we are never engaging only one of the building tasks. Thus, as in the example in the figure above, if you and I are discussing what it means to be a "good" mathematics teacher, beyond the meaning that we give to this topic we communicate a particular identity as a mathematics teacher in our interactions with particular people and discourses. This identity is communicated through the discourses we draw on in that particular interaction. We also favor different sign systems or kinds of knowledge, we make connections between ideas, and connect with various political beliefs in the discourses that we use. Since our recreations are not exact reproductions of past interactions, and as we interact with enough other people and institutions, the discourses shift and change. Moments of playfulness and vigilance, especially when others will play with us, can help us see the cracks in the dominant

discourses and work through these cracks.

Even though teachers are traditionally understood as the more powerful party in the classroom, the constraining power of discourses means that they do not operate freely. The norms of what it means to be a mathematics teacher are discursively and collectively enforced by students, other mathematics teachers, school administrators, teacher education programs, and society at large. In this way there is a constant normative pressure on mathematics teachers to take up the dominant discourses of mathematics education. In Fairclough's (2001) explanation there are at least three contexts that create this pressure. There is the immediate context (what it means to be a mathematics teacher or student in a particular classroom), an institutional context (what it means to be a mathematics teacher or student at a particular school), and a societal context (what it means to be a mathematics teacher or student in general). A mathematics teacher then needs to draw on discourses from all of these contexts in order to successfully re-create a recognizable although momentary mathematics teacher subject position. However, if there is a change in philosophy in any of these contexts the pressure can be greatly reduced or increased. For example, if a school adopts an explicitly social justice approach to education then a teacher would be supported in (or pressured to) politicize the teaching of mathematics.

Dominant Discourses

Dominance in discourses is one way in which power operates, accumulates and flows through discourses and becomes a means of limiting what is possible without the use of force. Dominant discourses are those discourses which exclude other ways of thinking and speaking by establishing an apparently static, universal truth. This casts

other discourses in the position of either not true, only partially true, or simply unimaginable. A discourse becomes dominant when it is widely accepted as common sense, and as a result, unquestionable. They can then be used to control, to some extent, the allowable content or topic of discussion; it can limit the possible ways of interacting, and it limits which roles are permissible in a given context (Fairclough, 2001). There are multiple discourses around mathematics that enable and constrain how teachers think about and practice mathematics education (Brown & McNamara, 2011). It is through these discourses that policy makers and others attempt to influence mathematics education. As an example of some of the many discourses that mathematics teacher candidates may engage with consider the following: the official, regulatory discourse of the Common Core State Standards which delineate what should be taught in various classes, the discourses in the curriculum materials based on those standards which specify what should be taught and sometimes how it should be taught, the discourses of the mentor teacher about how and what mathematics should be taught, the discourses of various mathematics teachers who have taught these teacher candidates through their K-12 education and college, the discourses of university methodology courses, the discourses of educational foundations courses about what education means, discourses of what it means to be a teacher and a student, and discourses of equity and social justice. Each of these discourses is multiple and contradictory and accesses varying levels of power depending on the specific context.

Even without direct supervision these discourses shape what happens in classrooms. Gutiérrez (2009) explains that a mathematics education research focus (which is also shaped by discourses) on the “achievement gap” in mathematics between

White students and students of color has changed the way teachers and schools understand what it means to teach mathematics and which students are most deserving of the teacher's time and attention. While there is little obvious evidence of policy changes to direct teacher's attention, teacher behavior is influenced by these discourses as they become dominant. When this research suggests that students scoring in the low-middle of an achievement test have the most potential to increase their scores, teachers face pressure to devote more time and energy to these students, potentially giving less attention to a significant portion of their class. This redirection of teacher time and energy occurs through self-regulation of behavior based out of a desire to support the school or department, out of a fear of getting caught doing or saying something outside what is perceived as part of mathematics education (dominant discourses), or even out of a belief that their efforts are what is best for the students. As the dominant discourses of mathematics education shifts so does the behavior of teachers (Gutiérrez, 2009). Thus even when no one is watching, the rules of these discourses are still followed.

Discourses of Whiteness

Thus far, I have only hinted at the role of discourses in gaining and maintaining systemic power. However, the pressure felt to conform to these discourses as well as the ways that the discourses place limits on what is possible to think and speak (and still be recognized as a mathematics teacher) maintain power structures. In the United States (and elsewhere) the dominant discourses (those which are represented as normal, common sense and unquestionable) are discourses of Whiteness. Whiteness Theory helps to uncover the ways in which discourses are used to maintain and promote White power and privilege. Whiteness Theory operates on the assumption that the lives of all people in

the U.S. in particular (but elsewhere as well) are racially structured, including the lives of White people (Frankenberg, 1993; Frye, 1992). Since historical mathematical achievement plays out on clearly racial lines in U.S. K-12 schools (Stinson, 2004), I operate from the assumption that race is a significant factor in defining mathematical success as traditionally measured within school mathematics.

Racial and gendered patterns are evident in almost any measure of mathematical achievement including the numbers of PhDs in mathematics granted to women or people of color (Martin, 2009), people of color and women who complete STEM careers (Metcalf, 2010), test scores on a variety of standardized tests and college entrance exams (Lubienski, 2002), and placement into the various K-12 mathematical tracks (Stinson, 2004). Often overlooked in these narrow measures of achievement are other racial and gender impacts of school mathematics. These include the literal erasure of the contributions of mathematicians of color to the historical development of mathematics (Almeida & Joseph, 2009; Joseph, 1994) and the ways in which students generally and women and students of color particularly are pushed to conform to the dominant discourses of mathematics education (Boaler & Greeno, 2000; Gutiérrez, 2012b;). It is important to note that, while White students generally are privileged in mathematics courses, there are many White students who are also excluded by the discourses of school mathematics. Further students are not the only ones caught up in these inequities. Walshaw (2013) demonstrates how an analysis of power through discourses shifts the focus from the "failings" of the individual teacher to an understanding of how systemic discourses shape teacher actions, often in ways that deflect attention away from social inequities. Both teachers and students are caught up by the dominant discourses of

mathematics education.

Despite these clear racial and gender patterns in mathematics education, most research on mathematics education has problematically only addressed race in mathematics by looking at students who are not White, usually in a way that frames students of color as inferior (Gutiérrez, 2002a; Martin, 2003; 2013); one problematic assumption in this research is that race is not a relevant factor in the education of White students. In this way the operation of the discourses of Whiteness is often invisible in mathematics education research. However, these dominant discourses exclude race talk from the discourses of mathematics education. I also suggest that the discourses of mathematics education overlap with the discourses of Whiteness in key ways which create a kind of doubling effect constraining the efforts of mathematics educators to teach for social justice.

In general, Whiteness is a power that is often invisible to White people (although it may be highly visible to people of color) and it is through this power that Whites maintain privileges and rights not granted to people of color (Frankenberg, 1993). While it is often easiest to think of Whiteness (and race in general) based on genetic indications (skin color, etc.) this can also be misleading. Whiteness is a set of beliefs, cultural values, and privileges that are maintained in part through the normative discourses of society. These discourses are used to construct White ideals and perspectives as universal norm and anything else as deviant. These dominant discourses of Whiteness then become the norm by which cultures and people are measured (Fairclough, 2001: Frankenberg, 1993), while simultaneously delegitimizing or hiding alternative discourses and perspectives. These discourses allow Whites to benefit from a racist society while maintaining an

appearance as nonracist, fair, rational, unbiased judge, and, bound by rule and principle, as above reproach (Berlak & Moyenda, 2001; Frye, 1992; Pratt, 1984). Key to the success of these discourses in maintaining White privilege is the lack of modifiers that mark the discourses as White (as opposed to various linguistic means of marking discourses as belonging to or representing a particular group, such as Black, women, or immigrants, for example). This lack of markers presents the discourses as universal, rather than White.

It is this particular claim to universality that maps onto key characteristics of school mathematics to the exclusion of social justice education thereby helping to maintain White educational privileges. To understand how this exclusion works I will use Whiteness Theory to analyze the ways in which the dominant discourses of mathematics education influences mathematics teacher candidates, particularly among mathematics teacher candidates who are otherwise committed to teaching for social justice. In light of the inequities within mathematics education Perreault's (1994) "blind embrace" of Whiteness is instructive. Perreault uses the term "blind embrace" to describe the ways in which discourses of Whiteness create a worldview that normalizes inequalities by allowing Whites to ignore history and dismiss current inequality:

> Much is lost in this blind embrace: both the facts of history, and the inequities of the present are erased—but the most glaring flaw is that the white speaker is once more claiming the right to define the parameters of 'humanity,' or 'universality.' (p. 235)

But this "blind embrace" requires that Whites actively not see, thus rather than blindness it is a discursively enabled shutting of the eyes. Not seeing history or inequity then allows Whites to determine what is fair and claim to be unbiased. Similarly mathematics education maintains an achievement gap perspective that holds White students as the

norm by which all others are judged (Gutiérrez, 2009). With the achievement gap, mathematics education does not see its role in the creation of the gap. Further, dominant mathematics ignores both the historical contributions of people of color to mathematics and the potential contributions of students of color to mathematics now, contributions that are made less possible by the inequities of school mathematics. This eyes shut view, which does not see either history or inequity, is strikingly similar to Walkerdine's (1990) description of mathematics where such contextual factors (including history and inequity) are stripped from a problem in order to give it mathematical significance. As a result mathematicians are left to define the extent of their mathematical universe without regard for political concerns.

As noted previously, an absolutist view of mathematics (Ernest, 1991) is currently dominant. This view of mathematics is characterized by a valorization of neutrality, abstraction, and formal symbolic language (Walkerdine, 1990). These characteristics coincide with the principal characteristics of Whiteness. Similarly Martin (2013) argues that mathematics education research is dominated by Whiteness in that it represents both White interests and White perspectives in defining what the problems and solutions worth exploring are within mathematics education. He also notes with some dismay that there have been minimal attempts to link mathematics and Whiteness in mathematics education research. Whiteness Theory provides a useful lens in this study because it provides tools and perspectives that can make visible the ways in which White privilege is maintained by the discourses of mathematics and the racialized effects of mathematics teaching and learning. These influences may have a norming effect on mathematics teachers, which serve to uphold the ways in which the education system privileges particular students.

Further, Whiteness Theory is essential to understand the push away from social justice and the pull towards neutrality and apparent apoliticality that seem to be valued so highly in both mathematics and Whiteness.

Whiteness Theory has potential to explain at least some of the reasons that mathematics teachers are particularly resistant to social justice efforts in the classroom. The discourses of Whiteness maintain White privilege with the façade that White privilege does not exist (Yoon, 2012). In a traditional mathematics classroom one of the ways in which teacher power is asserted is by evaluating the correctness of student responses. Thus when school mathematics is constructed to emphasize unambiguously "correct" answers (as opposed to open questions where "correctness" is relative) teachers are set up to evaluate these answers in a manner which increases their classroom power. While teacher discourses are generally used to reinforce the power of the teacher, for the mathematics teacher this effect is doubled since the discourses of mathematics education reinforce this power, in addition to the discourses of Whiteness that position Whites as judges. Thus the mathematics teacher is positioned as the ultimate judge. Since mathematics is decontextualized and abstract there appears to be little room for a biased judgment. There is a ready-made position for the teacher to judge students as either correct or incorrect and in a greater sense as mathematically capable or not. Similarly Kidder (1997), working with expatriate Whites in India, found that many fall into these ready-made positions of privilege and power. Many of these Whites came from a middle-class background in the United States and Europe, but when relocating to India, found the expected role for Whites includes the explicit exercise of power over their Indian hired servants.

The privileges these expatriate Whites enjoyed were both embraced and denied. Kidder (1997) found that, as these White women met, their discussions often revolved around the precariousness of their privileges and comparing their (relative) lack of privilege by pointing to other more privileged Whites. The privilege of these White families in India suggests racism (and colonialism and classism); thus in order to maintain an image as a "good" (i.e., not racist) White they had to find ways to justify and/or deny their privilege. This more current example is similar to the historical example presented by Anderson (1994) of the necessity of justifying slavery in light of U.S. ideals of equality. Justification occurs in these cases by positioning African Americans as inherently inferior, in the historic case, and by positioning Whites as not racist, for the expatriates. Both parts are key aspects of the creation and maintenance of racism. This particular kind of White goodness, with its accompanying justifications or denials of privilege, are particularly important to progressive Whites.

The specifics of this kind of White goodness will change and shift both over time and according to the specific context as (White) social values change. However, in general, goodness has consisted of some combination of "earned" wealth (see Lipsitz, 2006) and moral authority/superiority. This White moral authority is created through the portrayal of Whites as unbiased and not racist; this position is especially attractive for progressive Whites. In order to maintain this portrayal Whites must continually prove their "goodness" even if only to themselves. As Whites one way we can prove our "goodness" is by creating a hierarchy of racism or bias. As long as we can point to someone else as racist we can say that we are not racist like "that person." The effect is that we no longer feel the need to listen to those voices that call into question our

nonracism, even when that voice is our own. Through this arrogant maintenance of our goodness we reinscribe Whiteness. To be clear I am not suggesting that Whites should not try to be good. Rather that there is a particular way of being good that is more about proving our goodness than it is about making a real difference in terms of racism and inequity. Further this kind of goodness extends to antiracist academics who judge the attempts of younger/less-experienced scholars as naïve, which is a more sophisticated (implicit) way of saying "I am not racist like that person." (Thompson, 2003). Again this is not to suggest that we should not be critical of misguided antiracism, but that we not use criticality as a means to perform our own goodness.

I call this kind of goodness *normative goodness*, because of the ways that it pushes people to adopt dominant norms, sometimes at the expense of other values. While I am drawing the term from Whiteness theorists I want to separate normative goodness from a White racial identity, which could imply that people of color do not also face normative pressures. Instead I argue that both White teacher candidates and teacher candidates of color are pushed by normative goodness, especially as they come in contact with the dominant discourses of teacher preparation that present an idealized "good" mathematics teacher. This then becomes the standard by which teacher candidates (and their professors, supervisors, and mentor teachers) may measure their efforts to become teachers.

This normative goodness helps to maintain the authority of White people, especially insofar as we are able to portray ourselves as fair and unbiased. Teaching for social justice is at odds with this role of authority; it introduces contexts which the teacher may know less about than the students and which require both teacher and student

to make evaluations of various situations. This is a potential threat to the authority of the teacher since the teacher is required to recognize and validate the knowledge of the students. Because the role of teacher, especially that of mathematics teacher, is already molded with a discourse of authority (Gutiérrez, 2009), progressive teachers may find themselves taking on that authoritative role at times against their better judgment and desires. The teacher may use that authority in ways that promote White privilege and power by reenacting dominant discourses in the classroom. However, the teacher may use that authority instead to work alongside students to disrupt dominant discourses and for the promotion of socially just practices.

Alignment of Whiteness and School Mathematics

The discourses of Whiteness and school mathematics align in certain key areas. Currently the maintenance of White privilege requires the denial of the existence of racism and racial inequality in order to uphold the idea of normative goodness. This is achieved in part through the perpetuation of the following myths: meritocracy, neutrality, and colorblindness. There are, of course, many other ways in which White privilege is maintained. However, I highlight these three because of their important role and, especially for their potential for alignment with the discourses of school mathematics. While I will discuss each myth separately they are intricately interrelated and it is difficult to talk about one without the others. For example, meritocracy has to assume a system of judgment (to determine who has merit and who does not) in which color is not a factor since that would open up the potential for bias and destroy neutrality.

The myth of meritocracy goes something like this: anyone regardless of race, class, gender, etc. through hard work and good choices can achieve success at X (school,

work, mathematics, etc.). This myth perpetuates the normalization of race, class, and gender inequality by blaming the victim for their lack of success and exonerating the mostly White men who have benefitted from the (supposed) meritocracy. In a mathematics class this comes into play because of the decontextualization that is common in school mathematics. Since mathematics problems are abstract and stripped of contexts mathematics teachers may assume they give accurate measures (neutral judgments) of students' ability without regard for race, class, or gender. Rousseau and Tate (2003) found that mathematics teachers used mathematics to rationalize and explain away the racial patterns of failure in their mathematics classes, using discourses of hard work (or lack of it) to explain students' failure. In other words meritocracy suggests that these students failed because they lacked the necessary merits and not because of other potential factors such as poor teaching, a disengaging curriculum, or systemic racism in schools, among other factors. This reasoning, built on the perceived neutrality of school mathematics, normalizes the failure of these students by blaming them and deflects critique away from the teacher, school system, or society.

The myth (or idealization) of neutrality maintains that neutrality is both possible and desirable. Everyone, but especially authority figures, must maintain a neutral stance. If the authority figure's bias is revealed then he/she is labeled (racist, sexist, etc.) and removed from her/his position in order to maintain the appearance of neutrality, or rules are created to ensure that such bias does not affect decisions. If the status quo of a situation is challenged then the neutrality of the authority can be displayed to maintain the appearance of fairness that serves White privilege. While other fields have gatekeeping mechanisms, the perceived neutrality of school mathematics makes it an

ideal candidate as a gatekeeper for a number of high status disciplines as well as college entrance generally.

Mathematics' position as gatekeeper places mathematics teachers in this position of authority to deny or admit students. To maintain the White ideal of neutrality then mathematics teachers must appear neutral in their decisions. The common view of school mathematics as neutral enhances this ability, while teaching mathematics for social justice is a potential threat to this appearance of neutrality. Thus some of the status of mathematics teachers depends on perpetuating the idea of mathematics as neutral. The neutrality of mathematics teachers gives credence to the tests, grades, and other markers of success that determine student advancement in mathematics classes or in college.

The myth of colorblindness is the idea that it is both possible and desirable to not see the race of another person (Thompson, 1998). By actively not noticing color and then reporting to others how they or someone else didn't notice color Whites portray themselves as neutral, unbiased, and nonracist (i.e., "good"). This colorblind narrative then works to deny the experiences of people of color by denying the distinctness of their experience. From this perspective meaningful conversations about race are impossible to have since they would require noticing and talking about color. When race comes up in conversation it is sidelined or silenced and so race, and the inequalities connected to race, cannot be studied without violating the principle of colorblindness (Thompson, 1998). The perception of mathematics as universal and acultural makes it an ideal context for a colorblind perspective to thrive. Unlike other subject areas there is not recognizable Black mathematics, or Chicano mathematics, etc. in the same way that there is Black literature, Chicano history, or Indigenous art. However, the perception that mathematics

is acultural discourages teaching for social justice since to do so would violate colorblindness in general and the perception of mathematics as universal.

Within the education system mathematics teachers enjoy a certain status connected to mathematics. Generally mathematics teachers have an easier time finding teaching positions, have more choice in which teaching position they accept, and are more likely to have received funding for their degrees or other forms of financial incentives in becoming teachers. These privileges are directly tied to the discourses of mathematics and the perception of greater intelligence that accompanies mathematics. These privileges in addition to the myths explained above provide incentive to mathematics teachers to maintain the discourses of school mathematics, even if it comes at the expense of socially just teaching. However, this also requires progressive mathematics teachers to justify their failure to teach for social justice. While there are certainly many discourses that justify not teaching for social justice the following are those that I have found myself using or have heard other teachers use. In a mathematics context these could play out as teachers deny their own privilege, possibly pointing to the relatively low status of teachers. They may express an inability to act on social justice desires out of fear of offending students, parents, or others. Teachers may point to student misbehavior or ways in which students are responsible for their own failures or struggles. Finally, as noted by de Freitas (2008), mathematics teachers may disengage by suggesting that mathematics is just mathematics, without leaving room for human context and injustice. These match what Perreault (1994) found as common responses to a challenge of White privilege including denial of privilege, paralysis of fear or guilt, anger that is misdirected at the victim of privilege, and disengagement from discussion of

privilege. Kidder (1997) found similar denials of privilege among White expatriates in India.

Disrupting/Playing With Dominant Discourses

Part of learning to teach mathematics for social justice is learning to disrupt the dominant discourses, including normative goodness. Applebaum (2010) suggests that one way to work against this kind of normative goodness is what she terms vigilance. For Applebaum vigilance includes uncertainty, humility, and Foucauldian critique. Uncertainty includes a willingness to question our own certainty and what we think we know, especially about racism and the experiences of the other. Implicit in this is a requirement to not make final judgments (which suggests certainty). Humility means that we are "open to examining how our progressiveness might be oppressive in ways that we are not aware of" (p. 186). For our work in teaching mathematics for social justice this means that we critically examine the work that we do and see how even our promotion of social justice may reinscribe dominant discourses. Key to this kind of critique is to question our own frameworks of knowing, especially those things we think we know for certain; to ask ourselves what our assumptions make impossible for us to know or question. This cannot be done individually as it requires listening carefully to those who question what we think we know and who ask questions we would never think to ask.

Secondly, I borrow from Lugones (1987) the concept of *playfulness* as a potential way to disrupt the dominant discourses of mathematics education and Whiteness. While Lugones is specifically not writing from a poststructural perspective, I adapt this concept to fit within a poststructural frame. Lugones describes a kind of playfulness that creates opportunities for us to reform or re-create our worlds as well as to reform and re-create

our multiple selves within those worlds. While some worlds inhibit our playfulness, those where we can exist playfully allow us to create opportunities to change. The playfulness that Lugones describes can occur in any activity if we bring a playful attitude into the situation.

Before defining playfulness I need to first describe how Lugones (1987) understands "worlds" and "world-traveling." She draws these concepts from the experiences of people of color. Lugones explains that each of us inhabit multiple different worlds. For those who are powerful, navigating these worlds is relatively simple. For those who are not powerful, though, navigating these worlds can mean adopting/inhabiting different personas, some of which are restrictive or damaging to a sense of self. Lugones noted that in some of these worlds she could be playful and in others she could not. For her playfulness is uninhibited, unconcerned about rules, not taking yourself too seriously, a willingness to be a fool (or just to be wrong), and a willingness to be surprised. There are worlds/situations where we find it easier to be playful and others where we become rigid or defensive (not playful). For those of us in power we have to learn to be playful when traveling to other worlds, rather than imposing ourselves, which is manifest in many ways including the defensiveness that can arise in cross-race relations.

While Lugones' (1987) work implies some kind of preexistent core (even if multiple) self, which is inconsistent with the discursive, poststructural approach I am taking here, the concept of playfulness may still be useful. However, if we think of the "worlds" Lugones describes as various discourses, some of which promote playfulness while others do not, then playfulness can become a means of disrupting the norms of that

discourse. It is in playful moments that we can ask questions about what we think we know that might otherwise be absurd or even unthinkable. Playfulness can reveal the fractures and fissures of dominant discourses. Additionally, Lugones (1987) describes playfulness as a primarily individual trait (even though the example she gives involves another playful person). However, I find it useful to think of play as a social activity. While it may be necessary for an individual to take the first playful step, others help to set the conditions that make that possible. Further, in order to create a disruptive moment others have to respond with playfulness, otherwise the dominant discourses are simply reasserted. We cannot play alone, and disrupt a discourse.

Playfulness as described above is in many ways antiWhiteness, especially in moments where Whiteness could be challenged, and especially for progressive Whites. It is helpful in this description to think of the discourses of Whiteness as creating a set of rules or norms of what it means to be a "good," progressive White. Imagine, for a moment, a progressive White man in a classroom with other Whites and students of color. A question is brought up that suggests the possibility of racism (even if race is never explicitly mentioned), on his part. How does he respond? Can he possibly be playful in this moment? The chances are slim. For most White men in this situation who they think they are, as "a good White," has been challenged. He will likely now take himself very seriously as well as display increased concern about the (unwritten, unspoken) rules of cross-racial interaction. He is unwilling to be made a fool in terms of race relations (although he may be perfectly capable of this on other terms) and as a result unable to embrace the uncertainty, humility, and self-criticality necessary to listen to the other.

This is not to suggest that he should not take the implication of racism seriously, but instead that he should not take "himself" and his investments in being a "good White" too seriously. The example that Lugones (1987) uses to illustrate her conception of playfulness suggests a kind of joyful play. However, I suggest that there are other types of play that maintain the core ideas of her conception of playfulness. The key aspects of playfulness can also be present in a kind of serious play that may be more appropriate for the progressive White man in my scenario above. This playfulness can allow us to shift around, change and challenge the discourses of Whiteness and of school mathematics, even if only temporarily. For this to have lasting impact this play must be collective; we must both make and accept invitations to play.

Social Justice Mathematics

As a researcher and educator one of my goals in disrupting these dominant discourses is to create discursive space for imagining and implementing more socially just mathematics teaching practices with teacher candidates. To work against, play with, and decenter Whiteness in mathematics education requires a more robust and nuanced understanding of what it means to teach for social justice than the typical access (and sometimes achievement) focus advocated by the National Council of Teachers of Mathematics (NCTM) and other mainstream perspectives. To define what I mean by social justice mathematics I turn first to conceptions of social justice in education broadly and then more specifically within mathematics education.

Multiple authors note the lack of clear, agreed upon definitions of social justice in education, even though the term is now widely used (Grant & Agosto, 2008; Hackman, 2005; Hytten & Bettez, 2011; North, 2008). In an attempt to deepen the theoretical

foundation of social justice in education these same authors have reviewed the existing uses of social justice in education. Cochran-Smith, Shakman, Jong, Terrell, Barnatt, and McQuillan (2009) in making their case for the inclusion of social justice in teacher preparation state, "the bottom line of teaching is enhancing students' learning and their life chances by challenging the inequities of school and society" (p. 350). This definition, is useful because of its simplicity and the potential for bridging various perspectives on education. However, its necessary breadth limits its practical applicability. I turn to North's (2008) review which lays out three broad tensions to grapple with in understanding social justice in education. These tensions are between redistribution and recognition, macro- and microlevel issues, and knowledge and action.

In addressing these tensions North (2008) suggests that social justice must balance the potentially competing priorities of the redistribution of material resources and the need to recognize and embrace differences between groups. This tension should also recognize the fluid nature of group membership. Social justice will also need to address both macrolevel and microlevel (as well as in-between levels) of social injustice. Macro-level issues include broad national and global inequities and the policies that promote these inequities, while microlevel include the day to day happenings of a particular classroom and the needs of the students and teacher (both in and out of school) who come together in that class. The final tension suggests a need to balance learning about inequity (knowledge) and taking action. This tension recognizes the potential for damage if action is taken without adequate understanding of a situation as well as the fruitlessness of endless theorizing. To address social justice requires that we (and our students) engage in both learning and acting. In closing her article North (2008) suggests the need to continue

questioning and evaluating what social justice in education means, in order to avoid replacing one form of dominance with another. In this study, focused on the work of one small class of teacher candidates, our work tended to emphasize recognition, microlevel issues, and knowledge. This focus was in part due to the context of our work together as well as my perception of what was needed and possible for this group of teacher candidates. However, our discussions also addressed the redistribution of resources (including quality teaching), macrolevel issues (including discourses and standardized testing), and action (including what the teacher candidates could do in the process of becoming teachers)

Within mathematics education the theorizing of social justice has drawn heavily from Freirian critical pedagogy (Gustein, 2006). From this perspective social justice mathematics is understood as having mainly to do with curricular changes that involve the use of mathematics for social critique (Gutiérrez, 2015). This view is perhaps the most well developed within mathematics education and has led to increasing interest in social justice among mathematics educators. However, this view, which tends to dominate teachers' understandings of what social justice mathematics is, has, perhaps, contributed to the limited adoption of social justice mathematical practices among teachers. Additionally, it appears to give priority to certain aspects of (rather than maintaining balance between) the various tensions of social justice in education (North, 2008). In order to expand this understanding of social justice mathematics I turn to two concepts from Gutiérrez. These include Gutiérrez's (2012c) four dimensions of equity and Gutiérrez's (2009) description of an equity stance. Together these concepts encourage a broad and multiple understanding of social justice mathematics and are more

in-line with the tensions suggested by North (2008)

First, Gutiérrez (2012c) describes four dimensions of equity. These are access, achievement, identity, and power. She divides these into a dominant axis (access/achievement) and a critical axis (identity/power). Access deals with the resources that students have available to them, including technology and high-quality instruction. Achievement is measured in the traditional sense of grades and test scores. Both access and achievement generally leave the mathematics content relatively untouched. School mathematics is still the focus of instruction. However, this axis is necessary in order to provide students with the material resources and social capital to advance in the school system and to have the potential to impact the field of mathematics.

Attending to identity means providing opportunities for students to draw on their own linguistic and cultural resources, becoming better by their own standards, and coming to understand themselves and their world in relation to mathematics. Addressing power includes addressing whose voice matters in the mathematics classroom (authority is part of this), using mathematics for social critique, questioning the nature of mathematics and mathematical ways of knowing, and making mathematics more humanistic (Gutiérrez, 2012c). For a mathematical analogy this might be thought of as a coordinate plane[5] with each axis forming one of the axes of the coordinate plane. To deepen this analogy a complex coordinate plane[6] may be more useful with the dominant

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[5] This idea was first proposed when I gave a presentation to some colleagues. One of the mathematicians in the group asked whether the axes could be thought of as a coordinate plane. My initial response was that they could not, thinking that presenting the axes in this way implied a trade-off, for example, between access and achievement, but after additional thought, and if not carried too far, I believe it is a useful analogy.

[6] For those unfamiliar with the complex coordinate plane it functions similarly to the

representing the real axis and the critical representing the imaginary[7] axis. Just as imaginary numbers are invisible when operating within the realm of real numbers, the critical axis is hidden from a mainstream perspective of mathematics education.

Each instance of teaching can be thought of as a set of coordinates attending more or less to each of the four dimensions of equity, but over some lengthier period of time (a unit, a month, a semester) the teaching could attend to all four dimensions of equity. This framework is useful, in part, because it creates a wider view of what it means to teach mathematics for social justice. The image of the teacher regularly engaging students in the in-depth social critique lessons similar to the work of Gutstein (2006; 2009; 2012) is excellent, but it has come to be understood as the one and only way to teach social justice mathematics. This then becomes simultaneously too simple and too difficult. It is too simple in the sense that it leaves out other dimensions of equity as Gutiérrez's framework (2012c) makes plain and can risk perpetuating the idea of the (White) teacher as "lone hero" (Thompson, 2003). It is too difficult in the sense that teachers, especially young teachers, can feel inadequate if their ideas or lessons do not measure up to the examples given by Gutstein (2006). Gutstein does take up many of these issues. However, it seems that the best known aspects of his work are the excellent social critique lessons that he creates and teaches.

Gutiérrez's conception of equity is further enhanced by her suggestion that teachers develop an "equity stance" (2009). An equity stance is an off-balance stance that

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standard coordinate plane with the difference that one of the axes represents imaginary numbers instead of real numbers.

[7] The terms "real" and "imaginary" are mathematical terms that do not correspond to the ideas of existence, concreteness, or fantasy as they do outside of mathematics.

requires teachers to try to hold onto both sides of several binaries simultaneously. Maintaining this equity stance can help teachers avoid falling into the continuum of normative goodness. These binaries (explained more fully in Chapter 2) include knowing/not knowing your students, being in charge/not being in charge, and teaching mathematics/not teaching mathematics. The first part of each of these binaries (knowing your students, being in charge, and teaching mathematics) is a standard, common-sense concept of mathematics teacher preparation. The standard implication is that either you know your students or you do not, you are in charge or you are not, and that your responsibility is only to teach mathematics. However, as Gutiérrez (2009) pairs each one with its opposite the goal is to do each simultaneously. In stating and presenting these binaries Gutiérrez (2009) makes the limits of standard mathematics teacher preparation discourse more clear. Embracing this equity stance requires that mathematics teachers accept the uncertainty, humility and critique of vigilance (Applebaum, 2010)

Conclusion

Poststructural research has not commonly been applied to mathematics education research (Stinson & Bullock, 2012). However, poststructural understandings of the circulation of power through discourse can inform our understanding of the ways in which the discourses of mathematics education work to exclude social justice from mathematics education. Further, Whiteness Theory opens the potential to understand the racial dimensions of these discourses and the ways in which mathematics education serves to maintain White privilege in part through the progressive White desire for "goodness" that can exclude or minimize social justice. One potential means of working against this goodness is the idea of "playfulness" which may disrupt some of the rules of

White goodness. Finally, a more in depth and robust understanding of social justice that encourages vigilance in critiquing our own practice is necessary to trouble the dominant discourses of mathematics education.

CHAPTER 4

METHODS: CRITICAL POSTSTRUCTURAL DISCOURSE ANALYSIS

The site for the current study is an action research course I just finished teaching to a group of seven preservice mathematics teachers. These seven teachers took my course concurrently with their second semester of student-teaching, which provided the sites for their action research projects. The course met once weekly in the evenings. The first few course meetings focused on developing a shared understanding of what it means to be a mathematics teacher, the role of discourse in shaping our thoughts and actions, and on the role of social justice in mathematics classes. These classes were meant to challenge traditional thinking about teaching and to generate reflective thinking about the role of a teacher and the role of mathematics in our education system. These classes also prepared a foundation of social justice from which my students could select and investigate topics that address social justice in some way. Following these beginning classes (4 classes total), our focus shifted to explore what action research is, how it differs from other research traditions, and to the development of the preservice teachers' own action research projects. The selection of topics and development of action plans was a collaborative process through which we all offered feedback on research topics and methods (3 classes total). The remainder of the semester was spent with the students reporting on their progress on and adjustments to their projects and revisiting

mathematics teaching and social justice.

Participants

My interest in educational research grows out of my own experiences as a White high school mathematics teacher. I grew up immersed in and accepting of the dominant discourses that are all around us, especially discourses of Whiteness. I did not consider myself privileged and believed many of the myths of Whiteness, including meritocracy. I thought of racism as a problem of the past and had minimal interactions with people of color. Learning to speak Spanish and living for 2 years in southern Chile began to open my thinking to different perspectives on the world, racism, and discrimination. When I returned to the United States and reenrolled in college it was now with a goal of becoming a teacher. On completing a bachelor's degree in mathematics education and Spanish teaching as well as a master's degree in teaching English as a second language, I took my first teaching job at a public rural high school in Colorado. The high school had a student population of about 750 with 50% Latino students, mostly from Mexico and Central America. The school had a strong ESL program and was one of the few rural schools I found with a functioning sheltered mathematics program.

As a 1st-year teacher I taught all of the sheltered mathematics classes and continued teaching every sheltered mathematics class that was offered during my 4 years. Overall my teaching experience was very positive and the relationships that I developed with my students continue to evolve. However, I was also aware that my teaching was not what I wanted it to be. I identified then (and now) first as a teacher and secondly as a

mathematics[8] guy. What I found frustrating was the difficulty in teaching in a way that was so different from how I had been taught and in which I had little outside support. In addition, I felt that I could not see beyond the abstract, dominant mathematics in order to understand how to make the connections to students' lives that I felt were necessary. In this high school I witnessed first-hand the roles of race and class in the lives of students in our education system. There was a superficial harmony at the school between the wealthy White students and the working-class Latino students. However, there were clear divisions on race and class lines that determined which entrance to the school students used, what classes they took, which sports they participated in and supported, what cars they drove (or didn't drive), even where they parked their cars,[9] and where and if they went to college.

Mathematics classes were one of the key ways to maintain these divisions. Once these divisions were made (mostly in middle school) they were set. A student who began high school taking Algebra 1 would not make it to AP Calculus as a senior. As a new teacher I felt the pressure of maintaining my position as a teacher, and so supporting the school policies; I also wanted to better serve my students who were not being served by those same school policies. I wanted to teach mathematics in innovative and at times critical ways, but I felt the need to conform to traditional views of teaching mathematics.

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[8] I hesitate to use the term "mathematician" as I don't see myself that way. I have always been very good at school mathematics, but in my mind that is very different from being a mathematician.

[9] During my final year of teaching the school implemented a policy, over my weak protest, in which all parking stalls in the student lot were assigned to students, but to get a spot students had to present a driver's license, proof of insurance, and pay a fee. Since many of my students either did not have a license or could not afford the fee they had to park in the mud parking lot of the park across the street. School administrators did not see the policy as discriminatory.

Perhaps more importantly I saw that the school was not meeting the needs of my students in a number of ways. There was a stark contrast between the privileged educational experiences that I had and those of my students. The combination of these factors led me to pursue a PhD in education, in order to better understand the education of Latino students in the United States and to improve the teaching of mathematics.

Mathematics Teacher Candidates

The participants in this study were all students enrolled at our university in a program that will lead to a Master's degree in mathematics with teaching certification. These students are funded by Mathematics for America, which seeks to improve the mathematics education of U.S. secondary schools. As part of this process Mathematics for America recruits students with Bachelor's degrees in mathematics fields, funds their further education, and pairs them with accomplished mathematics teachers for their student teaching (Mathematics for America, 2013). This is a nontraditional program that leads to teacher certification by following a condensed version of the standard education curriculum that most teacher candidates experience. During the Fall 2013 academic semester I supervised these teacher candidates during their early months of student teaching. I also attended monthly MfA meetings and an MfA retreat with them. During the Spring 2014 semester I taught an action research class for these students as I continued supervision of their student teaching. In our various interactions several of the preservice teachers expressed an interest to me in learning to teach for social justice and four of these preservice teachers attended a practitioner conference on teaching mathematics for social justice in January in Los Angeles, CA. I consider each of these teachers to be capable and committed educators, who sincerely struggle with what it

means and how to teach mathematics for social justice. I enjoyed working with them and learned a great deal from them. This information is drawn from discussions that I had with these students (both in and out of class), their written work (particularly their mathematics educational history and what it means to be a teacher), my knowledge of their student-teaching contexts from supervision, and discussions with their mentor teachers.

Karl

Karl[10] was a 24-year-old student at the time he took my class. He grew up in a nearby state and came to our state in order to become a teacher. Karl describes his early mathematics experience as using a reform-based curriculum and states that he always loved mathematics. He particularly enjoyed the creative and problem-solving aspects of his classes, aspects that are typically not part of a traditional curriculum. He considered a career as a mathematician, but chose teaching in order to work with and prepare students for their own careers.

In his student-teaching Karl worked in a large, diverse public high school. Many of his students were English Language Learners and/or refugees. His mentor teacher was experienced and energetic. Much of his mentor's teaching was traditional, but he also regularly incorporated visuals and concrete objects to aid his students in gaining access to the content. Karl originally adopted many of these techniques, but as he questioned his authority he began creating many more opportunities for his students to discuss mathematics with each other and to pursue projects of their own interest.

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[10] All names are pseudonyms, most chosen by the students for themselves.

Jane

Jane's family is of Japanese ancestry, but immigrated to Mexico at some point. Her parents then moved to the United States shortly before her birth and Jane grew up speaking English and Spanish. Jane rarely referred to her cultural heritage unless asked specifically about it. She was 22 years old when she took my class and was part of the majority religion in our state. Jane enjoyed mathematics from an early age and did well throughout her schooling. When she found her friends turning to her for help with their mathematics she determined to pursue a career in teaching and later chose mathematics, because she enjoyed doing it and because of the value she believes it has for her students. Part of her goal in becoming a teacher is to help students achieve their own goals.

In her student-teaching placement Jane worked in a mathematics and science focused charter middle-school. The school was small and diverse. The teachers there had a great deal of autonomy and her mentor teacher regularly coordinated lessons with a science teacher as well. Jane's mentor teacher made her own curriculum built around projects and challenging hands-on problems. The students sat in pairs, discussed ideas with partners, presented and defended ideas to their classmates, and questioned each other's mathematical strategies daily. Jane adopted many of these same practices in her own teaching and helped explain to her classmates how to use some of these same strategies.

Gavin

Gavin was 27 at the time of my class and had grown up in the same state as our university. He was part of the majority religion of the area. He describes himself as liking mathematics because he got good grades in those classes. Gavin's desire to become a

teacher is connected to relationships that he developed with his own teachers, in particular a high school English teacher, and to be able to give back to society. He suggests that mathematics is a secondary aspect to his being a teacher and not his primary focus. Gavin also wants to work with students towards achieving their own goals.

Gavin student-taught in a White dominated, large public high school. His mentor teacher used a primarily traditional approach to teaching. Most of the class was spent in lecture, note taking, and individual work on homework assignments. Gavin often taught in this same manner, but began more regularly trying to incorporate group work, inquiry instruction, and student led projects. His mentor teacher encouraged this effort, but could offer little in terms of practical support.

Esperanza

Esperanza identifies as a Peruvian immigrant to the United States. She came with her family when she was 12 and grew up speaking Spanish and English and was 24 when she took my class. She attended schools in the same city as our university and attended classes in English as a Second Language speakers. Esperanza felt the lowered expectations that her teachers had of her because of her ESL label. Mathematics became a way for her to prove her capability to her teachers. Her goals as a teacher included a desire to provide her students with the opportunities that she felt she received from education, in particular for students who are in a situation similar to her own.

During her student-teaching she taught in a large, diverse public high school (coincidentally the same high school she had attended as a student). Classes in this high school were large (approximately 40 students in a class). For Esperanza as for all of the student-teachers this year the schools were transitioning to the Common Core standards.

At this school the district decided to implement a standard curriculum with standard assessments in each high school. Even though her mentor teacher taught in mostly traditional ways he was one of the only ones at the high school who was open to more inquiry-based teaching techniques. He supported Esperanza in her desires to teach in innovative ways and to reach out to ESL and students of color. However, he did not have the experience to offer her practical guidance in this area. Despite her status as a student-teacher, Esperanza took it upon herself to advocate for her students. She met with the principal and other teachers to find ways to better serve struggling ESL students. As she prepared to find a job Esperanza turned down multiple interviews, holding out to teach in a school where she would work with students of color and ELL students.

Lisa

Lisa provided very little demographic information. She was 26 at the time she took my class and is a White woman. She excelled at mathematics early on and enjoyed the status that she gained from being good at and helping her classmates with mathematics in high school. She delayed taking mathematics classes in high school, but after finishing her general education requirements she determined to major in mathematics. Lisa describes her college mathematics experience as very different from her early experience. She struggled in these classes and often did not dare to ask questions or talk to her professors in fear that they would discover her lack of mathematics ability. Eventually she successfully completed her degree. After working in restaurants for a year she returned to college to pursue a mathematics teaching master's degree and had a much better experience. In describing her desire to become a teacher Lisa gushed about the opportunity to work with young people and to influence them at

this key part of their lives. She particularly chose mathematics because of its perceived difficulty for these students. As she finished my class she expressed a clear view of teaching as a political profession.

In her student-teaching placement Lisa worked in a mathematics- and science-focused charter high-school. Lisa's classes were small and racially and socio-economically diverse. She worked with a very competent mentor who regularly used inquiry-based teaching methods and focused on traditional content. Lisa's mentor supported her social justice efforts even though she did not necessarily agree with them or have the ability to provide practical guidance in this regard. Lisa adopted many of the methods that her mentor teacher used, although Lisa placed much greater emphasis on developing relationships with and understanding her students' perspectives.

Stella

Stella, at the time of my class, was a 22-year-old White woman. She grew up in an affluent neighborhood in the same city as our university and eventually took a job in the same high school that she had attended as a student. She excelled at mathematics in middle and high school. She determined to become a mathematics teacher, because mathematics was her favorite subject and she enjoyed working with kids. Stella talks about being a teacher who can guide students both in gaining deep understanding of and ownership of mathematics and in their life choices.

Stella worked in the same school as Lisa, although she worked primarily with freshmen (Lisa worked with juniors). Stella worked with a competent and experienced mentor teacher who followed her reform-based textbook very closely. Stella also followed the textbook closely, although she began developing more of her own materials

later in the semester. Stella excelled at finding ways to engage her students in meaningful and in-depth mathematical discussions.

Jeff

Jeff, at 48, was the oldest of the group. He was from a neighboring state, but had lived in our state for much of his adult life. He had prior degrees in psychology and physics and had worked for a number of years in business before returning to college to become a teacher. Jeff did very well in his mathematics classes and was comfortable with the traditional style of teaching those classes. However, his experiences with his own children in mathematics classes and observing classmates led him to believe that these methods were not the most effective for many students, and this is part of what led him to be a mathematics teacher.

Jeff completed his student-teaching at a large public high school. His mentor teacher was experienced, but was also adapting to her 1st year at this school. This, coupled with a number of personal life issues that she faced and with a lack of Common Core aligned curricular materials for her grade level, meant that Jeff did not get the support he felt he needed. Most of her time was spent developing curriculum and there was little time for collaboration. Jeff was also commuting from over an hour away. As a result he was very concerned about what he considered the basics of being a mathematics teacher which included developing mathematically focused lessons that were accessible to students and classroom management. While he expressed appreciation for social justice issues, he also suggested that these would best be taken up later in his teaching. Table 1 summarizes participant information.

The course I taught these teacher candidates is one of the final requirements for

Table 1. Summary of participant characteristics

Chosen Pseudonym	Self-selected Race	Gender	Age	Background Information	Student-Teaching Context
Karl	White	Male	24	From Washington; loves mathematics and the outdoors	Taught in a traditional public high school. Worked with many ESL and refugee students. Mentor teacher was very supportive; also traditional and capable.
Plain Jane	Culturally diverse[a]	Female	22	From state of study; Spanish-English bilingual, member of dominant local religion	Taught in a diverse mathematics/science focused charter (middle and high) school. Mentor teacher was supportive and masterful in creating nontraditional mathematical lessons and engaging students in mathematical conversations.
Gavin	None (White)	Male	27	From state of study; speaks Spanish as a second language, member of dominant local religion	Taught in a majority White, very large public high school. Mentor teacher was mostly traditional and very supportive.
Esperanza	Peruvian/Latina	Female	24	From Peru, grew up in state of study; Spanish-English bilingual	Taught in a large public high school with restrictive curriculum policies. Worked with and advocated for ESL students. Mentor teacher was mostly traditional and supportive.
None (Lisa)	None (White)	Female	26	From state of study; previously worked in schools in a nonteaching capacity	Taught in a diverse mathematics/science focused charter high school. Mentor teacher regularly used reform methods and was very supportive.
Stella	None (White)	Female	22	From city of study; religious	Taught in a diverse mathematics/science focused charter high school. Mentor teacher regularly used reform methods (strictly following a reform textbook) and was very supportive.
None (Jeff)	None (White)	Male	48	From Wyoming; has worked previously in business, physics, and psychology	Taught in a large public high school. Due to various circumstances Jeff did not feel he got all the support he needed from his mentor and school. Teaching was mostly traditional.
Teacher	White	Male	34	From state of study; speaks Spanish as a second language, member of dominant local religion	Taught (prior to study) in a diverse public high school. Worked primarily with recent immigrant Latino students.

[a] Although Jane chose "Plain Jane" as her pseudonym I use Jane throughout to be more consistent with the other pseudonyms. Her reference here to "culturally diverse" is a reference to her Japanese ancestors, who later immigrated to Mexico. Her parents immigrated to the United States where she was born. She grew up speaking Spanish and English.

their certification as teachers. As a group these students were committed to doing the best they could for their students and while they struggled at times with all of the responsibilities involved in becoming a teacher, they also maintained a commitment to equitable teaching practices. Some of them are working to form a local group of teachers committed to teaching for social justice.

Data Collection

Pedagogy: Action Research and PAR

Since the data collection for this study will come from the action research course I taught in Spring 2013, some explanation of my understanding of action research is necessary. I was working with these teacher candidates in the process of becoming mathematics teachers with the authority and power that goes along with their position. In terms of social justice and equity my concern in this work was principally for the current and future students that these teacher candidates did and will teach. During that semester the teacher candidates engaged in AR projects in their own classrooms. I draw on the broad action research approach defined by Greenwood and Levin (2007) and Zeichner (2001). Additionally I view the importance of relationships in participatory action research, (Cahill, 2007; Cahill & Torre, 2007; Doucet & Mauthner, 2012; Tuck, 2009), the attention to power relations in PAR (Maguire, 2001), and the focus on participant, rather than researcher, interests (McIntyre, 2008) as crucial to my own work with these teacher candidates.

I categorize the action research (hereafter AR) approach of Greenwood and Levin (2007) to as critical. While Hagevik, Aydeniz, and Rowell (2012) note that critical AR has often been disconnected from the more practical AR that often takes place in

education, this disconnect is unnecessary and for this study a critical focus is essential to align with the social justice approach to education that I advocate. Greenwood and Levin (2007) describe an idealized form of AR that often will require adjustment according to the context. They along with Reason and Bradbury (2001) define AR as a participatory democratic process in which researcher and participants work together as "equals," but with different forms of knowledge and expertise.

Zeichner (2001) suggests that action research follow a spiraling process of planning, action, observation, and reflection, which fits with the philosophy of Greenwood and Levin. I follow a similar spiraling process. For this class the planning stage included a series of reflective journal entries by the teacher candidates that were designed to draw their attention to relevant research topics within their own teaching. With feedback both from myself and their peers the teacher candidates selected topics and gathered relevant research articles. Following planning the teacher candidates put their plans into practice in their teaching. During and after implementation the teacher candidates gathered data on the relative success of their plans. These data included accounts of what happened during the class, student statements collected during or after the lessons, samples of student work, and other evidence on how their projects were progressing.

In conjunction with and during data collection the teacher candidates created regular reports for their colleagues, who in turn provided feedback on how to adjust the plan for future implementation. Following this feedback, the preservice teachers made further attempts to implement their plans. A single implementation, data collection, and reflection cycle was designed to take approximately 2 weeks. However, state mandated

testing and spring break required making some adjustments to the timing. The teacher candidates completed this cycle three times during the semester culminating in a final written report and a poster presentation. Both Greenwood and Levin (2007) and Zeichner (2001) recommend that this process be collaborative and work towards social change. Collaboration both with me and with peers is built into this process and I encouraged students to select projects that work towards social change and asked them to explain the connections in their work to social justice.

Relationships

I view relationships as central to my work both as a teacher and as a researcher. As a former high school mathematics teacher I respect the work of secondary school teachers and hold this respect as central to my relationship with the teacher candidates that I supervised. As a consequence I work to avoid research by "us" and about "them" (Cahill & Torre, 2007), in this case by me and about these teacher candidates. Instead I strove to work with these teachers as we all worked to improve our practice and strive for greater social justice in the classroom. At the beginning of this class I had already worked for about 4 months with this group of students. I supervised their student-teaching experience and attended workshops, dinners, and other MfA sponsored events where we had multiple opportunities to talk not just about their teaching, but also about their lives outside of school. I worked to solidify those relationship from the 1st day of class where, during our first activity, I tried to "flip the script" and encouraged them to ask me whatever questions they would like (not about the class, we would do this later) in order to counter the typical classroom experience where the teacher gets to ask all of the questions. I continued this work of relationship building throughout the semester.

Cahill (2007) frames the need to consider relationships in research in ethical terms, which go beyond what is required by institutional review boards (IRB) to work alongside of a particular group in achieving some benefit for that group. Cahill explains that these relationships can establish trust and that it is this trust that ultimately leads to participants working with a researcher and not the legalistic promises of confidentiality found in IRB consent documents. However, this trust must go beyond the kind of trust shown by the students and women who worked with McIntyre (2008) who simply stated that they trusted her to represent them appropriately. This trust that McIntyre cites appears to flow only in one direction, from the participants to the researcher, and does not require that the researcher also trust the participants to speak back to the work and to take a hand in how they and their work is represented. The problem with this kind of trust is that it could be developed using a form of "faked friendship" in which the researcher appears to take an interest in the participants, but only with the intention of getting participants to trust the researcher (Duncombe & Jesssop, 2012). In contrast, a relationship in which the trust flows both ways (researcher to participant and participant to researcher) is more difficult to establish. This two-way trust is difficult because it requires the researcher to take risks, and requires the researcher to trust that the participants are capable and prepare them to speak back to the work in meaningful ways. In my case I had to trust that the teacher candidates would consent to participate (I did not know who had consented until after all data collection was complete) and trust that they would engage with the class in ways that would provide usable data.

In my work with these preservice mathematics teachers I found the establishment of two-way trusting relationship to be difficult. There were times when I was successful

in this attempt and others when I was not. I encouraged the teacher candidates to choose topics that would allow them to make some improvement in the class and teaching, but to also consider the benefits for their own students. To create these relationships Cahill (2007) emphasizes the need to listen to the needs and desires of those you work with. Tuck (2009) further explains the importance of listening as an indigenous "aural" (as opposed to "oral") tradition that is essential for collective work. I created opportunities in class for students to bring up topics that were relevant to them, to ask their own questions, and I read their reflection journals about their teaching to "listen" to the challenges that they were facing as teachers in order to help them identify potential action research topics that would benefit them and their students in the process of becoming teachers.

While the ultimate decision of what topic to pursue for further research was theirs, I wonder about times where my own influence and perspective served to silence students' perspectives and desires, because they "trusted" me. In fact, when I asked my students what biographical information they would like included in this study more than one responded that they trusted me to fill it in for them. However, there were also times in class when students would disagree with my perspective (always in a nonconfrontational way) or present an alternative perspective. Further in the student journals, (where I put most of my efforts in guiding them to an action research topic) students responded to my comments in complex ways, sometimes accepting what I said, sometimes ignoring it (possibly because they didn't see it), sometimes expanding on my thoughts, or taking the dialogue in a different direction. The idea of a two-way trusting relationship is neither constant nor is it the same with each student. Tuck (2009) further explains that

relationships are not just between the researcher and participants, but also relationships among the participants. There seem to be times when it is there and others when it is not. Part of my responsibility is to be more aware of when it is not and to make adjustments.

In our class I encouraged the students to both support and critique each other. They regularly provided detailed feedback to each other, asked sometimes difficult questions, and provided material support when possible. When one student brought up the need to address the problem of teacher authority in mathematics classes, other students took up this idea and looked at it in their own work. I also looked at my own use of authority and pointed out to students how I was using it and ways I was trying to mitigate the effects of that authority. This sense of "we're all in this together" was further facilitated by the cohort model of the MfA program which meant that these students took most of their classes together and attended regular meetings and workshops as a group. The students themselves established helpful (critical, if necessary) relationships as an entire group with no one member left out. They cheered for each other's successes and provided redirection as needed. To the extent possible I included myself with this and cheered along with the students and critiqued my own practice as I was asking them to critique theirs. Including myself as a participant caused a minor stir with IRB, which discouraged this lack of researcher "objectivity;" however this difficulty was resolved.

Unseen Relationships

Doucet and Mauthner (2012) add that researchers must also consider "unseen relationships," which has pushed me to think beyond just the relationship between my students and I. By "unseen relationships" they refer to the various organizations that we communicate with. In my case, there are multiple relationships, which have influenced

the work I have done with my students. For example, I have a relationship with the teacher education program at the university, which employs me to teach the class and supervise the teacher candidates. As a consequence I feel a responsibility to them to teach certain topics or to do certain things in the classroom that I may or may not do otherwise (mock interviews, professional portfolios, etc.). When this was the case I tried to make the teacher candidates aware of where such requirements are coming from, in order to maintain my relationship with my students, while also fairly representing the other side. I also have a relationship to the field of teacher education research, in particular mathematics teacher education, and to the work done by researchers on social justice in mathematics education. At times my desire to represent social justice mathematics to the teacher candidates got in the way of the relationships I hoped to have with the teacher candidates.

More important than my unseen professional relationships are the unseen relationships between the work that I do with the teacher candidates and their current and former students. I feel a deep responsibility to these middle and high school students to prepare teachers who will give them a quality education, which will prepare them both for access and achievement in college and beyond as well as a critical view of the world facilitated by mathematical understandings. This desire has pushed me to discuss equity and social justice more regularly in my class, to encourage my students to address social justice directly, and to require my students to justify their projects in terms of social justice. I believe that it is important that I not push my ideas and aims onto my students, but that to deny or hide these desires is disingenuous and damaging to the transparent and open relationships that I hope to establish.

Power Relations

Part of establishing meaningful, trusting relationships with my students means paying attention to how power and authority flow through the class. Working in this context as teacher, researcher, and supervisor I take on subject positions that are typically constructed with authority over students and participants. However, to work effectively, ethically, and justly in my context I needed to find ways to "flatten" these power relationships (Maguire, 2001). Since this is also a research project there is an additional layer of authority that would not normally be present in a class. From the very beginning I have been open with my students about what I am studying and what I intend to do with the information I collect (McIntyre, 2008). While this is important it is only a piece of acknowledging the role of power. Kesby, Kindon, and Pain (2007) provide an important reminder that, from a poststructural perspective, power is always present in relationships but that it can have productive effects. This productive power can be used in positive ways. Part of the work that I do in forming relationships with my students serves to lessen the power differential between my students and me. There were also times when the male students in the class dominated the discussion and I took steps to choose topics that the female students were more comfortable participating in and making sure that all of the students had opportunities to comment and ask questions.

I noticed in my own teaching that I set the agenda for classes and often made sure that I got the last word in a discussion. While it may be appropriate for me to set the agenda sometimes, there are other times when students could take on more of this responsibility. For example, in some of the classes I asked students to come up with their own questions about the readings and then the entire class focused on discussions of their

questions. I have tried to speak less and listen more in these discussions to prevent my voice and perspectives from dominating all of the time. There have been times when I have worked out with students when it makes the most sense to have an assignment due or the order in which to have different classes. After one of the teacher candidates decided to investigate his own use of authority, I also tried to make my use of authority a discussion point in class, and at times the teacher candidates pointed out when they felt I was acting too authoritarian. I believe to not acknowledge the differences in power and authority that accompany our traditional class roles is hypocritical and damaging to the relationships that I worked to form with students. While these steps have not erased power differences, they do help to "flatten" those relationships somewhat.

Participant Interests

Paying attention to and honoring participant interests is an essential part of establishing meaningful, trusting relationships. McIntyre (2008) states that researchers should be "scrupulous in their efforts to give primacy to participants' goals" (p. 12). While in most research the researcher's goals are often the only ones taken into consideration (or even known), in PAR the participants' goals should be the focus of the group's (including the researcher) efforts, with the researcher's goals secondary. In this project this was a tricky balance. Out of respect to my obligations to my students' current and future students, I maintained an explicit social justice orientation in my teaching and in the projects I encouraged students to take on. At the same time, in maintaining consistency with my desire to honor my students' goals, I wanted them to choose their action research focus. In an attempt to strike this balance I assigned students to maintain regular journals to reflect on what they were learning in my class, what was happening in

their classes, and their thoughts about their teaching. Through comments on their journals I engaged in dialogue with my students and highlighted potential action research topics that they were discussing in their journals. I especially tried to draw attention to those that could take a social justice turn. The ideas for the projects came from the students and ultimately they chose their topics, but I tried to influence them towards a social justice focus. Additionally, I assigned my students to explain how their topic connects to social justice in education.

Action Research Projects

In the end my students selected topics that included using formative assessment to meet student needs, increasing student authority in the classroom, making effective use of cooperative learning for all students, homework as a means to improve learning outcomes for marginalized students, engaging students effectively (including cultural relevance), and getting to know struggling students. While these topics are similar to traditional topics of interest in mathematics education, each also has a social justice aim in improving the outcomes and class experiences of historically marginalized students.

Data Sources

The action research course provided the data sources for each question. Each class was recorded and transcribed, resulting in over 25 hours of recorded and transcribed data. In addition the teacher candidates' written work, primarily reflection journals and final report, as well as my reflection journal were also data sources. Each research question drew on slightly different components of the class. I will describe each data set in connection to the question it answered. Question 1 is meant to first identify what

discourses the teacher candidates and I used in discussing their own teaching of mathematics and the place of social justice in it. These topics were discussed most often with the whole class during the opening several classes as we discussed discourses and social justice teaching.

The 2nd question goes further to look at how the discourses around mathematics and the discourses around social justice come into contact with each other. To respond to this question I looked for data on apparent conflicts, support, indifference, etc. between these discourses in our recorded discussions. These conflicts came up most frequently in our discussions of discourse and mathematics teaching for social justice, as well as in their oral presentations of their action research projects. In these reports the experience itself is less important than how the teacher candidates talk about their experiences. Britzman (1994) notes, "The primary category of analysis is the discourse of experience rather than experience itself. Here, experience does not 'tell' us who we are, what we see, or even how to act; we are the tellers of experience" (p. 56). It is as teachers report on their experiences that they make the connections between what happened and what they think about what happened. The final question gets at how the teacher candidates and I tried to manage the sometimes competing discourses of mathematics and social justice. For this question I turned primarily to my written reflections and the teacher candidates' written work.

Data Analysis

In line with the definitions of discourse and ideology (given by Gee, 2005; 2011; Fairclough, 2001) used in the theory section I used methods of Critical Discourse Analysis to analyze the data given, and drew on Whiteness Theory to understand the role

of these discourses in the exclusion of social justice. I draw on this work to do a broad analysis. As I reviewed our class transcripts and the teacher candidates' written work I selected those sections where there was evidence of, or potential for, addressing issues of social justice within mathematics education. I did not analyze sections of transcript that were only about mathematics or only about social justice, because they did not directly address the questions I was asking in this study. I analyzed, but did not include, transcripts which seemed to duplicate aspects of the analysis that I have presented in later chapters, to avoid redundancy.

Part of the purpose of this study is to increase the visibility of the beliefs and values behind the dominant discourses that we (the preservice teachers and I) used to discuss mathematics and social justice. Gee explains that "the goal of discourse analysis is to render even Discourses with which we are familiar 'strange'" (2005, p. 102). By rendering the common discourses "strange" we will then be more able to see the ideologies underlying these discourses, why they exist, and whose purposes they serve. I approached this process using Gee's (2005) seven "building tasks." These building tasks are the various things people do by using Discourses. According to Gee (2005), in speaking and being in the world people build significance, activities, identities, relationships, politics, connections, and significance for signs systems. Of these building tasks significance, identity, relationships, politics, and connections were the most significant for understanding the issues around discourses that I explored here. Consider the following brief transcript from one of our class discussions on how schools and teachers can affect students' access to mathematics courses.

433. Gavin: But even if they have those kinds of options
434. what kind of students are going to want to come in during summer

435. to catch up on something [they're not necessarily sure they want to do.
436. Stella: [Yeah.
437. Stella: Yeah and they have to pay money to do it
438. and their parents have to drive them there.
439. Gavin: It restricts the access I think.
440. Teacher: Yeah I was just going to say is that really access?

Significance

We build significance by what we draw attention to (or away from). In this portion of the discussion Gavin makes the group of students that he is talking about significant by using the phrase "kind of" (l. 434) rather than simply "students." This draws attention to these students, distinguishes these students from students generally, and makes them more significant.

Identity

One way that we build identity (or subject positions) through discourse is by who is positioned as agent (the subject of the sentence) and in what way. All of Gavin's and Stella's comments focus on the students. They build identities for students as uninterested in mathematics (l. 435) and unwilling or unable to confront obstacles to come to summer school (ll. 437-438). Of course Stella and Gavin are also building their own identities as knowledgeable about students and school systems. In contrast, in my statement (l. 440), I am the agent. In this way I draw attention to myself and present an identity as a critic.

Relationships

We build relationships both with the other speakers present and with others who may not be present. By using "yeah" (ll. 436, 437, 440) Stella and I both signal a relationship with Gavin that is supportive of him and what he is saying. In particular

Stella's use of "yeah and" (l. 437) mark her comment as an extension of Gavin's thought and suggest a relationship of mutual agreement.

Politics

We build politics through discourse based on the implications of these discourses for the distribution of social goods. Our discussion above, with its focus on who has access to the social good of mathematics education, is explicitly political, but this explicit politics is not necessary for a conversation to have political implications. Prior to the discussion above Stella had brought up a summer school program that would allow students to move from general to honors mathematics. Gavin's opening comment is in response to this and suggests that the program does little to increase access. This aligns him as someone who advocates increased access, possibly based on untracked classes. My comment carries similar implications. Stella's comment more directly suggests that the social good of mathematics education should not be connected to money or transportation, which also possibly supports untracked classes.

Connections

We build connections by creating links between ideas and objects. In this case Stella links issues of finance (l. 437) and transportation (l. 438) to the issue of access to mathematics classes. This link is not a natural part of the definition of access, however, this link may be a common one in some discourses and not in others. In making this link Stella frames the idea of access in a way that has specific political implications (see above).

Even in a brief transcript we build many different things through the discourses

that we use. While all of these building tasks may be present in any given selection of text, some will be more relevant to a particular analysis than others. Additionally some of the tasks will be more prominent than others in different selections of text. As I present my analyses I generally draw on only one or two of the tasks at a time in order to present a more coherent argument. Fairclough (2001) adds that a discourse analyst should look at the experiential (how an experience is represented), relational (how relationships are constructed), expressive (the author's evaluation), and connective (how parts of discourse are connected) aspects of discourse. In particular a researcher should pay attention to how these aspects of discourse constrain and involve contents, relations, and subjects. This process allows for an analysis of how power is taken up, used, and influences people in a particular situation.

Critical Discourse Analysis draws on poststructural understandings of discourse and the circulation of power (Rogers, Malancharuvil-Berkes, Mosley, Hui, & Joseph, 2005) In particular that as speakers use discourse, they construct the objects and subject positions that they speak about. But even these objects and subject positions are multiple and changing; they are continually negotiated and re-created. Thus my analysis portrays the multiplicity of possibilities in the presented data, names the dominant discourses and how they constrain what we can think, and how our use of discourse reflects the binaries of these discourses. I have also framed these discourses within Whiteness Theory to analyze the ways in which these discourses maintain White privilege and how we begin to rework and undo some of these discourses in order to understand teaching mathematics for social justice. Specifically I use Whiteness Theory to identify the binaries set up by

our discourses and then revisit my analysis to identify the binaries that I present in my analysis.

CHAPTER 5

ABSTRACT MATHEMATICS AND THE "GOOD" MATHEMATICS TEACHER

School mathematics is largely perceived as fair and neutral, and, as a result as a potential equalizer of racial, gendered, and other social inequities. However, it frequently serves to maintain these same inequities. These inequities are manifest along race, class, and gender lines, as well as along other markers of difference. Mathematics education may be more resistant to efforts that promote social justice and equity than other subject areas (de Freitas, 2008; Gutstein, 2006; Walshaw, 2013). One reason for this is the dominance of abstract mathematics in mathematics education. The discourses related to abstract mathematics deflect attention from social inequities and the concerns and circumstances of the students in the mathematics classroom. The concept of abstract mathematics falls within a binary of pure and applied mathematics. Within this binary pure mathematics (and pure mathematicians) is granted higher status. While school mathematics arguably is neither pure nor applied, the influence of this discursive binary pressures mathematics teachers to also privilege pure mathematics. This in turn privileges mathematics content which claims to be universal and decontextualized and those students who learn to do mathematics in universal ways. In this way school mathematical ability is equated with overall intelligence as students who are successful at school mathematics are considered more intelligent than students who are not. School

mathematics then becomes a means of dividing students, purportedly according to their intelligence. As these divisions are created the discourses around school mathematics deflect attention away from the roles of teacher, administrators, and mathematics itself in creating these divisions (Skovsmose & Valero, 2001). Instead the focus is on the individual students and their apparent intelligence (or lack), work ethic (or lack), or other individual characteristics.

These discourses, which privilege particular ways of doing mathematics and particular students, overlap with discourses of Whiteness to do the work of justifying the inequitable outcomes that are the norm in mathematics classes across the United States. Anderson (1994) explains that myths about the inferiority of Africans allowed White Americans to reconcile the evils of slavery with the ideals of the American Revolution and the Constitution. In a similar way current myths about the intellectual inferiority of multiple groups (i.e., that African Americans, Latina/os, girls, etc. are not as good at mathematics as White males) allow a reconciliation of the clear inequities in the United States with the ideals of a free, democratic, post-civil-rights, postracial society (Yoon, 2012).

The intellectual hierarchy perpetuated through school mathematics is a key component of the achievement gap. The achievement gap myth is particularly powerful and rests on a racialized (and gendered) hierarchy of intelligence whereby all groups are compared (mostly unfavorably) to White males. In mathematics this hierarchy rests on the dominance of abstract mathematics. The perception of abstract mathematics as neutral, apolitical, and acultural make it an ideal vehicle for creating and maintaining divisions among students, since it does not carry the "taint" of subjectivity that is

commonly associated with other school subjects (social studies, language arts, etc.) and is perceived as more "pure" than science. This allows those who witness and enforce these divisions (mathematics teachers, school administrators) to justify their decision making since it was based in the "purity" that is school mathematics (Skovsmose & Valero, 2001).

In later chapters I delve into how teacher candidates take up discourses of abstract mathematics in order to create divisions among students and to structure teacher-student relationships, as well as how teacher candidates work against these discourses. In this chapter I focus on how we (the teacher candidates and I) challenge and maintain the discourses of abstract mathematics as they apply to mathematics teaching. Key to this analysis is an understanding of how our desire to be "good" mathematics teachers led us to reassert the dominance of abstract mathematics even as we tried to critique this same dominance. As we attempted to disrupt some of the dominant discourses of abstract mathematics we were unable to, in part, because of the pressure we felt to maintain the dominance of abstract mathematics. At other times by approaching issues around mathematics education playfully we could temporarily break out of the dominant discourses to consider new possibilities.

A key feature of discourses of Whiteness is how we use them to "embody and accept contradictions and hypocrisies" (Yoon, 2012, p. 590). Further, discourses of Whiteness have a way of appearing, especially to the speaker, to promote antiracism, while they simultaneously support Whiteness in other forms. This occurs, for example, when a teacher critiques one aspect of traditional mathematics education (like the tendency of abstract mathematics to exclude), while simultaneously calling into question

the ability of students to appreciate an alternative curriculum. Such an argument might go something like this "Constructivist teaching techniques (or group work or projects, etc.) are better, but I can't do them with X group of students because they are too disruptive (or unprepared, etc.)." These arguments have been used to justify the creation of lower-track mathematics courses that are then mostly filled with students of color. These arguments are also used to justify the rejection of constructivist teaching techniques for students of color. In this chapter I present examples of how we at times were caught in similar discourses as well as times when we temporarily disrupted some of these discourses. Yoon (2012) notes that, for teachers, "a white-centered professional culture disables race (or more accurately antiracist) talk" (p. 589). In large part this chapter is about how an abstract mathematics-centered discourse disallows social justice discourses through our desires to see ourselves as "good" mathematics teachers.

In analyzing these moments of discussion and written work I draw on Gee's (2005) building tasks. In particular I use the building tasks of "Significance" (what the speaker means and makes relevant) and "Activities" (what the speaker intends to do through discourse). Later chapters use others of the building tasks, but these two are the most relevant to show how we discuss abstract mathematics and social justice as well as the moments of contradiction and slippage. Where possible I also connect our discussions to the underlying beliefs and values that we draw on.

As a class we studied various perspectives on social justice and equity; however, the students most regularly referred to the equity framework developed by Gutiérrez (2012c). I will also use this framework in order to make sense of students' statements regarding social justice. In this framework Gutiérrez explains four aspects of equity.

These include a dominant axis, access and achievement, and a critical access, identity and power. Within this framework the dominance of abstract mathematics is problematic for multiple reasons including its inaccessibility for many students. For similar reasons it is difficult for many students to achieve high levels of success. These two problems (access and achievement) combined with the understanding of abstract mathematics as the only (or at least the purest) form of mathematics positions mathematics as an ideal tool with which to sort students into tracks and hierarchies of intelligence.

In terms of identity abstract mathematics is alienating to many, particularly students of color and women. In fact Gutiérrez (2012b) explains that many women and students of color have to leave aspects of their identity behind in order to succeed in more advanced mathematics classes. In discursive terms this means that the discourses of school mathematics push away some of the other discourses these students regularly use. For these students this alienation occurs in part because of the historical erasure of the significant contributions of people of color and of women to the development of mathematics and to the exclusion of other forms of mathematics. But alienation also occurs as the contributions of current female students and students of color are not taken as seriously as the contributions of White, male students. Finally, in terms of power, school mathematics encourages a classroom focus on correct, decontextualized answers and on the teacher as knower, both of which assert the authority of the teacher and the status quo of social inequity, both in and out of school. This chapter is organized into two sections. The first section explores those moments in class when we momentarily disrupted some of the dominant discourses of mathematics education. The second section focuses more specifically on those discourses that limited our ability to understand

teaching mathematics for social justice.

Playfully Disrupting Abstract Discourses

Playing With Discourse

The utility of Lugones' (1987) conception of playfulness became apparent as I attempted a reanalysis of our class discussions during week 4, especially in contrast to our class in week 3. In particular the idea that playfulness can create opportunities for us to rewrite the rules of our mathematics education world helps explain some of what happened during week 4. However, since I am jumping ahead to the 4th week of the semester I will first summarize what led up this class.

During our first class meeting we spent some time getting to know each other and understanding the purpose of the class. We also discussed how action research differs from traditional ideas of research. We did not hold class during the 2nd week of the semester. Instead I traveled with four of the teacher candidates (Jane, Esperanza, Stella, and Lisa) to Los Angeles, California for the Creating Balance in an Unjust World: Conference on Mathematics Education and Social Justice 2014. The travel and conference costs for the four teacher candidates was paid by MfA on the condition that they present what they learned to the rest of MfA when they returned. All seven of the teacher candidates wrote about and participated in an online discussion about what it means to be a teacher and how mathematics has led to various privileges during their school experience.[11] During the conference we observed high school mathematics classes in the Los Angeles school district, participated in various workshops (given by practicing

[11] For specifics on these and other assignments as well as a summary of class readings see the Appendix.

teachers), and had lengthy (sometimes passionate) conversations about mathematics teaching and social justice. The students who attended this conference, especially Esperanza and Stella, repeatedly referred back to key moments from this conference as reshaping their approach to teaching mathematics.

In preparation for our week 3 class I wanted the teacher candidates to begin imagining mathematics in different ways in order to reveal some of how current conceptions of mathematics structure the world we live in. To do this I asked them to create a time system that uses a different base[12] from the current mix of bases that make up our current time system and to write about how changing the time system would change the way they experienced time. I also asked them to view a video[13] on base-8, *The Simpsons*, and the mathematical structure of the Simpsons' universe. Each of these activities was set up to encourage the teacher candidates to play with these mathematical ideas.

However, as Lugones (1987) points out playfulness is not about the activity (playing with bases), but about the attitude we bring to the activity. While I had high hopes for these activities generating meaningful discussions about how our world is structured by mathematics, the results were disappointing. I had expected that the teacher candidates, in part because of their advanced mathematical knowledge, would think deeply about how mathematics shapes the world. Each of the students came to class

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[12] Briefly, the base of a number system determines how a number system is organized. Our principal modern number system is base-10 meaning that we use 10 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9), and then add digits to the right to increase the value in powers of 10 (i.e. $1 = 10^0$, $10 = 10^1$, $100 = 10^2$, and so on). Another common base is binary (base-2), which is used in computer programming.

[13] http://io9.com/why-dont-the-simpsons-live-in-a-base-8-world-1456312130

prepared and having completed what I asked them to do. They all presented mathematically sound ideas for time systems and worlds that used different bases and then all came to the conclusion that even in a world with a 10-hour day living in that world would only be superficially different from our experience of living in our current world.

At least at the time that appeared to be the consensus from the class. Their responses were decidedly not playful. One of the female teacher candidates presented her ideas first. She was followed by each of the three male teacher candidates presenting their ideas and taking over the discussion, which focused on the mathematical details of their new time systems. This degree of the male teacher candidates dominating portions of the discussion was rare. It is possible that some of the female teacher candidates, who did not speak, had more nuanced responses. Unfortunately, I did not use the audio recorder correctly for this class and as a result I have no record of what was said, beyond some brief notes taken after the class. The disappointment of having lost the recording overpowered the disappointment of the superficial levels of engagement with the class activities, but I moved on, and did not think much more about this class until I later read Lugones' (1987) work as part of my analysis of week 4.

The week 4 class was in stark contrast to this experience. In preparation for this class I asked the students to read selections from Gee's *An Introduction to Discourse Analysis: Theory and Method* (2nd Edition) and Fairclough's *Language and Power.* During class I showed the students a video clip[14] explaining the history and development of the telephone number pad, to illustrate how one discourse can come to dominate and

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[14] http://www.numberphile.com/videos/keypad_layout.html

exclude alternatives. My intention in this class was to open up discussion about the way discourses around mathematics, teaching, and mathematics teaching influence us (as mathematics teachers) to teach and interact in certain (fairly consistent) ways as a group and how these discourses also influence the way students understand the nature of mathematics. For many of these teacher candidates this was their first exposure to discourse, dominant discourse, and ideology (at least as explained by Fairclough). In many respects I consider this a break-through class. The preservice teachers engaged with these ideas readily and in depth, especially in exploring the ways that discourse shapes our experiences and thinking. In later classes they regularly referred back to this class and what they learned from our discussions. The language of discourse and ideology appeared to have allowed them to articulate more critical views around mathematics and mathematics education.

Prior to the selection transcribed below we had been discussing what students considered to be dominant discourses in mathematics teaching. Included in their suggestions were:

- the emphasis on algebra in secondary mathematics
- assessment and grading practices that prioritize one correct answer
- the linearity of mathematical knowledge development
- the use of lecture and direct instruction as primary teaching techniques
- teaching specific algorithms for students to memorize

They also recognized that there is variation in how dominant these ideas are and that local contexts can push teachers in different directions. The final point about teaching students specific algorithms that are meant to be memorized led to a lengthy discussion.

Stella introduced this idea and then gave an example of her cooperating teacher requiring students to solve systems of equations using the equal values method. The specifics of this method are not important at this point; instead the focus is on requiring students to use any particular algorithm as opposed to allowing them to use a method of their choice.

Following Stella's comment I generalized the question and posed it to the rest of the class. "Why do we or why do mathematics teachers sometimes put on there [on tests or homework] solve it this way? Solve it using the quadratic formula, solve it using completing the square, solve it using factoring." The topic that Stella brought up and my following question were not part of my plan for the class that day. This question prompted approximately 30 minutes of discussion by the teacher candidates where I participated only occasionally and minimally. By deviating from the planned lesson and following the lead of my students I was opening myself up to the possibility of being surprised, and I was surprised, by the ideas the teacher candidates had. Prior to this moment I was following the common teacher class interaction where the teacher initiates a question, the student (or students) responds, and the teacher evaluates this response (Initiation, Response, Evaluation/Feedback, or IRE/F, Watson & Young, 1986; Wagner, Herbel-Eisenmann, & Choppin, 2012). The evaluation is a kind of judgment on the part of the teacher. This evaluation or judgment is an attempt by the teacher in this situation to control the content and acceptable participation in the discussion. This judgment discourages playfulness. In response to Stella's comment I suspended this cycle momentarily, by reforming her comment as a question for the class to consider. After a few comments to keep the idea going and make sure everyone was clear I could let the discussion go, the students took over, and I stopped evaluating their comments.

The most interesting surprise as the discussion developed was when Karl brought up a moment in his own teaching when he presented a formal, algebraic proof of the quadratic formula to one of his classes. Karl's comment came after several of the teacher candidates had debated the merits of requiring students to use particular algorithms or giving them choices. They had focused specifically on solving quadratic equations using the quadratic formula, completing the square, and factoring. Esperanza, in particular, had talked about the value of completing the square and of deriving the quadratic formula with her students. As she finished her comment the discussion began to die down and Karl took this moment to pose a question to the class that sparked a new discussion. In framing his question Karl shifted the focus from specific mathematical practices to the potential effects of those practices on students.

My initial analysis of this transcript focused on the ways in which the language of discourse analysis created a space for critique of mathematics educational practices. As I returned to it, and especially by placing it in contrast to the failures of week 3, the idea of playfulness provided a richer explanation of what happened between these two classes. The teacher candidates appear to have responded to my activities in week 3 from within their mathematics world. Within this world they take mathematics and its accompanying rules (too) seriously, which did not allow them to approach the task with a playful attitude and resulted in superficial analysis. Their investments of time, money, and identity in mathematics were too important to allow the possibility of surprise or of being a fool. Further the three successive responses from only the male students framed the discussion as exclusively mathematical, not experiential. In contrast, the activities around discourse were not connected to their mathematics world or to their teacher world (which

they have also invested in). While they have also invested in developing a sense of themselves as students, this investment was not tied to the particular content (discourses, in this case) and they were likely more at ease in the student world (since by this point in their education they are experts at the norms of being a student). This greater comfort level with the student world, the discussion of a new content area (discourses), and the temporary suspension of typical classroom discourse (the IRE cycle and its inherent judgment) may have allowed them to use the concept of discourse to critique mathematics educational practices in a playful way. In this case the teacher candidates engage in a kind of serious play that allows them to question some of the fundamental aspects of the utility of abstraction in secondary mathematics classes.

Karl initiates the conversation below by challenging a common mathematics educational practice, in this case the derivation of the quadratic formula by completing the square.[15] This practice, traditionally and as explained by Karl, is exemplified by a formal lecture with a heavy emphasis on the manipulation of mathematical symbols. Students are not expected to be able to replicate the practice. This type of lesson is often justified in terms of showing students where a particular idea comes from, not historically, but mathematically. While there are times when this mathematical development can be done in a way that is accessible to at least some of the students, in

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$$ax^2 + bx + c = 0$$
$$x^2 + \tfrac{b}{a}x + \tfrac{c}{a} = 0$$
$$x^2 + \tfrac{b}{a}x + \tfrac{b^2}{4a^2} + (\tfrac{c}{a} - \tfrac{b^2}{4a^2}) = 0$$
$$(x + \tfrac{b}{2a})^2 = \tfrac{b^2}{4a^2} - \tfrac{c}{a}$$
$$(x + \tfrac{b}{2a})^2 = \tfrac{b^2 - 4ac}{4a^2}$$
$$x + \tfrac{b}{2a} = \tfrac{\pm\sqrt{b^2 - 4ac}}{2a}$$
$$x = \tfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

[15] It looks like this, from http://www.these.com/mathematics_and_computer_science/algebra/quadratic_equations/quadratic_formula_derivation_files/image002.gif

this particular case it was not. From my experience with this kind of practice the few students who follow the presentation end up impressed by the teacher and baffled by the mathematics. This can be an ego boost to the teacher, but it may be of little mathematical value. Karl calls this practice into question by asking us to consider how it might affect the students and teacher-student relationships. Implicitly, he is making an argument about whose voice matters in the classroom. While we had not yet read Gutiérrez's (2012c) equity framework, Karl is critiquing this practice because of the power relations it set up in his classroom and the undue deference that he received as a result. In presenting this to me and his peers he is taking some very real risks including looking like a fool in front of his supervisor (me) and his classmates.

1. Karl: So um I want to I guess pose this question to everybody so um . .
2. I think like we in sec II we derived the quadratic formula
3. and I would say that like my students
4. we kind of started out together doing something different
5. and then it was kind of more like
6. alright here's Mr. Karl taking the reins and like doing everything
7. and you know what percentage of kids in secondary II
8. do you think understand what's going on
9. when you actually prove the quadratic formula?
10. How beneficial is it
11. and would it be more beneficial to do something
12. that more students would understand?
13. Cuz I feel like like the formal proof of something like that
14. goes over 95% of my kids' heads
15. and it's probably maybe even detrimental
16. and that's kind of my thought after teaching doing it one time
17. and what do you guys think about that?

Lines 18-41 ((the students clarify what Karl means))

41. Karl: I have never seen someone teach the quadratic formula in a way
42. that a reasonable amount of students benefit from it.
43. Teacher: Can I pause the discussion here for just a second?
44. multiple: Yeah. Mhmm.
45. Teacher: I really like where I think this is going
46. so I want to keep this going
47. um this is this is the question ((I indicate a question on the board, "Who benefits?", that I wrote on the board while listening to Karl))

48. that comes to my mind when I heard your statement.
49. Do you want to go here or continue this for a minute?
50. Karl: Continue this.
51. Teacher: Ok. Carry on.
52. Jane: So I have a question with this whole like just give me the formula
53. like what does that do for the students to just give them a formula
54. that they don't really know where it's coming from
55. they don't know like all they're doing is just give me the quadratic formula
56. so I can just start plugging stuff in
57. great like what does that do for them cuz
58. Teacher: Can I can I pause you for a minute? So let me write that down.
59. Um cuz I think that is going to take us away from this
60. but I want to have this discussion too. K
61. so can we pause that and continue this for is that ok?

In posing this question Karl talks about a moment in his own teaching in which he presented an abstract explanation (the derivation of the quadratic formula l. 2) that his secondary students did not understand. He notes in describing the moment that while they (teacher and student) started out together, he ended up "doing everything" (l. 6). To him this reified his position as knower in problematic ways and placed his voice above all others in the classroom. As a student-teacher he could have questioned his ability to teach the concept well or he could have questioned his students' ability to understand. Instead Karl questions this mathematics educational practice and implicitly frames the question as an issue of social justice by placing the question in terms of "benefit" (ll. 10-11) and "detriment" (l. 15) to the students. This framing sets up a binary where a lesson is either beneficial or detrimental and hides the possibility that a lesson could be both (either beneficial to some and detrimental to others or beneficial in some ways and detrimental in other ways). Karl's framing of his question in these terms allowed us to consider this practice in new ways. Typically with the kind of formal, algebraic proof that he referenced the focus is entirely on the teacher and the work on the board. Karl shifts the focus to ask what the student experiences and how that affects the student. In this case,

Karl is questioning the appropriateness of teacher authority, as expressed through abstract mathematics, working against the creation of a socially just classroom environment, in which students' voices and ideas are heard and valued. It becomes clear in later discussions and from Karl's written work that his primary concern is how his voice excludes other perspectives from being heard in the classroom. What Karl has noticed is how the discourses of abstract mathematics as taken up by the teacher can serve as a means of excluding student perspectives on mathematics from the classroom.

This moment represents a time of serious play. Note that for this discussion it is Karl's question, not mine, which initiates and prompts the remainder of the discussion. In this way Karl continues the disruption of the standard, teacher dominated class discussion. While I do some work to bring the rest of the teacher candidates on board with Karl's question (in the omitted section and ll. 58-61) notice that I also defer to Karl in determining the direction of the discussion (ll. 43-49). Following Karl's question I take on a less visible role in the discussion, and both he and I (temporarily) demonstrate a willingness to suspend the rules that typically govern classroom interactions. For the most part I allow the students to develop the discussion as they choose.

As Karl sets the stage for his questions and portrays himself taking over the lesson in his class (l. 6) he adopts a mocking tone. He made this negative evaluation of his teaching confidently, demonstrating a willingness to not take himself too seriously. He is unconcerned with being right and also unconcerned about portraying himself as a competent teacher to his classmates or to me. He directs his question not to me, but to his classmates. By leaving the question open to his classmates he is inviting them to "play" along and allowing for the potential for surprise.

After the teacher candidates discuss the nature of this teaching moment (lecture style, algebraic manipulations, lots of symbols, etc.), I step in (ll. 43-49) to recognize the importance of the discussion and present an alternate discussion direction. Karl bids to continue the discussion in the same direction, but Jane (ll. 52-57) also proposes a new direction (questioning the common practice of giving students formulas, without understanding), which I ask her to put aside until we complete the current discussion. Jane's comment is also a challenge to a traditional practice in mathematics classes and like Karl she frames it in terms of benefit to students ("what does that do for students" l. 53, "what does that do for them" l. 57). They are both using this discussion to challenge the traditional use of abstract mathematics, which is a significant change from the first class, discussed above. Importantly, it centers benefit to students in a way that was not part of our previous discussions about mathematics teaching.

It appears that Jane's question draws on some of the discourses of constructivist reforms of mathematics education. These perspectives were consistent with both her methods course and her student-teaching site. Constructivism commonly critiques traditional mathematics practices for the overemphasis on algorithms, as Jane does with multiple references to giving students a formula (ll. 52, 53, 55) and the idea of "plugging stuff in" (l. 56). Instead Jane suggests that students need to "know where it's coming from" (l. 54). This idea of concept development and student understanding is also a point of emphasis from a constructivist perspective. While Karl's emphasis on student understanding (ll. 8, 12, 14) may also draw on constructivist discourses (and he does more explicitly later in this discussion), it is not clear at this point.

The question Karl asks is an important one for mathematics teachers to take up

and discuss. Specifically when he asks "would it be more beneficial to do something that more students would understand?" (ll. 12-13), this question could lead to a reimagining of or playing with different possibilities for school mathematics including the possibility of social justice mathematics. However, constructivist discourses (if Karl is drawing on these discourses) do not disrupt the dominance of abstract mathematics. Instead these discourses leave the content of school mathematics largely unchanged, focusing instead on the pedagogy and how that mathematics content is presented to students. Because of my own preference for constructivist methods at times the ways these constructivist discourses center abstract mathematics escapes my notice.

However, challenging these kinds of practices within the context of abstract mathematics is difficult and Esperanza brings up a counterargument while maintaining the stance of focusing on students in this next section. Notice how she frames her argument in terms of "exposure" and teacher "obligation." In contrast to Karl, she makes an argument that focuses on the access students have to high-level mathematics. Embedded, and unquestioned, in her argument is the assumption that abstract mathematics is automatically high-level.

Being Good

In the transcript that follows Esperanza refers to a moment in her teaching when she also presented a derivation of the quadratic formula. However, there were key differences in their two presentations. Esperanza's presentation was developed from a presentation to MfA by Henri Picciotto,[16] which she refers to in lines 65-68. This

[16] See this page from Henri Picciotto's website for an idea of what this kind of derivation

approach to the derivation of the quadratic formula uses more graphs (in the algebraic sense) and is intended to be a constructivist lesson in which the students "discover" the quadratic formula. When this approach was presented to us as if we were students there was a sense of playfulness amongst the group, in part because it was novel and challenging. This playfulness is the enjoyment (l. 65) that Esperanza refers to. So while Karl was recreating a traditional practice, Esperanza was modifying the same practice in ways that were more consistent with the constructivist ideals of her methods class.

This modification represents an investment of time and thought that Esperanza put into this practice that Karl likely did not. This is also an investment by Esperanza in positioning herself as a "good" mathematics teacher, since she is trying out the practices she had been taught to use. These multiple investments may have positioned Esperanza to defend her lesson, and by extension the practice of deriving the quadratic formula, which inhibited her ability to accept Karl's invitation to playfully challenge this practice. This does not in any way invalidate her critique; instead it demonstrates the ways in which dominant discourses (as ideas of what it means to be a "good" teacher) can disallow questioning our practices.

62. Esperanza: I just wanted to say
63. I honestly will tell you I mean the majority of the secondary II students
64. were not ready to even see the algebra behind deriving the formula
65. and as much as honestly I enjoyed it
66. and I think all of us did
67. when we did it with Henri because we are math teachers and we've played with it
68. and it definitely did not come out in the way that it came out in our meeting
69. but at the same time one of the reasons why we chose to do this was
70. because the students are not
71. and have not been exposed this way of learning mathematics
72. even though it's over their heads

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looks like http://www.mathedpage.org/parabolas/graphical.pdf

73. and I did like we paused and we're like ok we're taking over
74. cuz this is like not moving
75. cuz students are not so familiar with the algebra
76. using the variables and everything got very messy
77. but at the same time I felt not that everybody did
78. I felt but I felt like some students to an extent might have appreciated
79. and we as teachers are in the obligation to show them what
80. to show them where things come from
81. to not take things at face value
82. to learn that math can also be taught in those ways
83. even though it could have gone over their heads when we did it
84. and that's most what happened

Esperanza begins this defense by conceding Karl's point that most students "were not ready" (l. 63) and also recognizing that there is a certain enjoyment (l. 65) that she, as a mathematics teacher, got out of deriving the formula. This positions her to present a counter to Karl's question without upsetting the friendly dynamic of the class. She begins her argument with "but" (l. 69), positioning it in opposition to the idea that there may be detriment to students from this kind of abstract mathematical presentation. She frames her work as an explicit decision on her part (l. 69) because students "have not been exposed" (l. 71) to it before. In this way she, like Karl, frames her argument as a social justice issue.

Esperanza is connecting her argument to the mainstream equity discourse of mathematics education on the importance of providing all students with access to high-level mathematics. This connection is made more explicit later, but here, she is attempting to frame the argument as an issue of social justice, as she understands it. In the process the abstract mathematics seems to force a conflict between attention to classroom power relations and student access. This conflict comes as Esperanza attempts to balance being a "good" mathematics teacher, by following the recommendations of her methods class (using constructivist methods) and by using an idea from a professional

development workshop put on by MfA, with her own desire (and possibly mine) to increase the access that students (particularly language minority students) have to higher-level mathematics and discovery learning. Additionally she links what it means to be a mathematics teacher to enjoying abstract mathematical problems (ll. 65-67) noting that "all of us did [enjoy it] . . . because we are math teachers." This signals her personal commitment to this particular lesson, because it grew out of an experience that she enjoyed (l. 65). Both what it means to be a "good" mathematics teacher and her methods course were explicitly focused on abstract mathematics as was her lesson on the quadratic formula. The resulting conflict stands out in her choice to construct the benefit of this lesson as "exposure" (as opposed to "learning" or "understanding"). Exposure does not imply learning or understanding on the part of the students, nor does it imply the kind of active engagement or discovery that is the goal of constructivist lessons. Instead it seems to be a kind of showing to demonstrate "where [the quadratic formula] comes from" (l. 80), mathematically. In this way, despite the significant differences in approach, her lesson becomes very similar in effect on the students to the lesson Karl described and questioned earlier.

In some ways her perspective engages both the reality of an education system (which tracks students based on abstract mathematical ability) and the pressure that teachers feel based on curriculum and standards, both of which focus on abstract mathematics. In response to this pressure to be a "good" mathematics teacher she turned the argument away from attention to power relations and toward student needs for advancement in the education system. She is saying that there is mathematical thinking that schools have withheld from students which this activity can provide. This justifies

her overt use of authority in "taking over" (l. 73) the lesson, instead of maintaining the discovery focus of her constructivist lesson. Her taking over is framed as necessary while for Karl taking over is what signaled to him that there might be a problem with what he was doing. Esperanza also used "exposure" to justify this teaching practice even though not all of her students understood (ll. 72, 77, 78). Since this kind of mathematical thinking is something that students' need, "teachers are in the obligation" (l. 79) to provide it to students. Esperanza then links her argument to Jane's prior statement (ll. 52-57), because students need to know "where things come from" (l. 81), also drawing on constructivist discourses.

As Esperanza continues explaining her perspective on deriving the quadratic formula notice the continued emphasis on exposure and her framing of this practice as "good for" (l. 88) students.

85. but I think exposing them to that particular teaching method
86. of them learning about it is good for them
87. not that they understand it
88. but it's good for them to realize that math is not only about
89. ok here's a formula just plug it in like Jane was saying right
90. I think it to an extent the way you present the formula
91. we present it in a way where like we were going to derive it
92. this is really difficult even like us struggles
93. like which one do I do
94. but the thing is you understanding that this is coming from somewhere
95. that you know the graph of the parabola
96. rather than just like magically this formula works
97. and it gives you the zeros for the quadratics
98. so not so much on deriving the formula
99. but actually giving students the opportunity to see that level of math
100. or that way of teaching math is what I care [((unintelligible))
Lines 101-108 ((there is some further clarification of Karl's question))[17]

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[17] Here (and in later transcripts) for the sake of brevity I have skipped a portion of our discussion because it was not relevant to this analysis. However, to alert the reader that I have skipped a section and to provide contextual information I include a brief summary

109. Karl: Mine was more like
110. I think the quadratic formula is on a completely other level
111. and I'm wondering because like a lot
112. I guess like I'm adding to my question maybe this is bad
113. but like a lot of tasks I think you can structure in a way
114. so that students come up with a formula and they're like oh yeah that's great
115. like the surface area of a cube is this
116. and that's great because I can actually like think about this
117. the quadratic formula is like out in space.
118. Esperanza: Yes. Yes.

She repeats the importance of exposure (l. 85) and evaluates that as "good for them [students]" (l. 86), because it shows them that "this is coming from somewhere" (l. 94) and is not "magic" (l. 96). She concludes this portion of her argument by focusing on the "opportunity" (l. 99) provided by this kind of teaching. She has framed her argument in terms of social justice by focusing on students' needs and opportunities and by challenging the "magical" quality that abstract mathematics is perceived to have. Esperanza's argument centers on students' needs and teacher's obligations. In doing so she has set up a situation where a teacher (herself) is required to teach abstract mathematics. As Karl clarifies his thinking in light of Esperanza's argument he agrees with her point about the value of constructivist teaching (ll. 113-116), but separates out the quadratic formula (l. 117). Esperanza agrees with this point. To clarify his question Karl takes on the voice of a student (ll. 114-116) who has "come up with a formula" and, in this way, points to the benefit to the student who now recognizes a new ability to "think about" mathematics. Here Karl more clearly draws on constructive discourses with his emphasis on students rediscovering mathematical ideas (l. 114). However, Karl maintains his position that the derivation of the quadratic formula is beyond the majority

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of the skipped lines.

of students ("completely other level" l. 110; "out in space" l. 117). In this way Karl manages the discussion in order to agree with the access portion of Esperanza's position, while also maintaining the idea that presenting the derivation of the quadratic formula is potentially detrimental to students.

In a sense what has happened is that Karl is arguing for attention to power relationships while Esperanza is arguing for student access to abstract mathematics. Karl is suggesting that teachers should consider teaching in less authoritative ways, while Esperanza suggests that teachers are obligated to provide access to abstract mathematics, even if this means teaching in more authoritative ways. Both are drawing on different aspects of Gutiérrez's (2012c) equity framework in ways that illustrate the complexity of applying socially just teaching practices within the context of abstract mathematics. This is not to suggest that the two are incompatible, but for these teacher candidates it seems that abstract mathematics is forcing a choice between access and power relations.

This conflict appears in part because the ideal of a "good" mathematics teacher focuses exclusively on abstract mathematics, while a focus on power relations calls the value of abstract mathematics into question. Abstract mathematics is presented as neutral, suggesting that there are no power relations to challenge. Part of this perceived incompatibility comes because of the ways that discourses of mathematics education link teacher authority (based on mathematical knowledge) to rigorous mathematics and student voice with lowered standards. In this way a binary is set up which requires a mathematics teacher to choose between rigorous mathematics (Esperanza's argument) and greater emphasis on student perspectives (Karl's argument). The dominant discourse is that student contributions cannot be as mathematically rigorous as the teachers, at least

within abstract mathematics. This is especially true for students of color and women. Within this discourse then Karl and Esperanza seem to feel that they must choose between preparing students for advanced mathematics classes (access) and opening classroom time and space for greater student involvement in learning. This binary hides the possibility of teaching a lesson that addresses access and power.

While I will revisit Esperanza's argument as it continues to evolve, it is important to note that both she and Karl are constructing arguments around the abstract nature of school mathematics. Both implicitly link their arguments to equity (one for access and one about power relationships) even though they are arguing from different perspectives. While Esperanza does not say so explicitly, part of the disagreement may stem from differences in how each originally presented the derivation of the quadratic formula. Where Karl presented his in a lecture style, Esperanza, at least initially, presented hers geometrically (more visually) and hers was meant to be student driven even though it was still very abstract. So in this sense she had planned for greater student involvement than Karl had. However, her students were unable to make the connections as she had planned and for this reason she took over the lesson.

The point here is not to debate the merits of how Karl or Esperanza approached presenting the derivation of the quadratic formula to their students. Instead we need to understand why Karl challenges his own practice, while Esperanza defends hers. Karl challenges the practice as potentially detrimental to students, because of the reification of teacher authority. Esperanza defends it because of the potential she sees in difficult mathematics and constructivist methods to grant students access to more advanced mathematics and further education. Esperanza is trying very hard to be a "good"

mathematics teacher both by meeting the expectations of the various people involved in her becoming a mathematics teacher and by meeting her own expectations to prepare students like herself for mathematical success. She wants to use good teaching practices (using recommended methods), she wants to be a good MfA member (using a practice MfA presented), and she wants to be a good teacher by meeting her students' needs. This was also an activity that she enjoyed doing herself and may have hoped that her students would also enjoy. From a teacher education perspective these are all qualities that we want her to develop. However, the pressure she senses to do all of these things as a student-teacher may have positioned her (at least in this moment) as unable to play in a way that will permit her to question some of the ideals of mathematics education and to question the potential detriment or benefit of these "good" practices.

While not in the text itself, race and gender likely play an important role in how each frames the argument differently. Karl as a White male may feel his authority in ways that Esperanza as Latina does not. Additionally as a second language learner and a student of color Esperanza may be particularly in-tune with the tendency for schools and teachers to hold lower expectations for these students, while Karl as a White male working with refugee and ELL students likely does not sense these low-expectations in the same way. In Esperanza's written reflections about discourses and her responses to some questions I posed to her in her journal, she explored the challenges that she faced because of the various ways she felt the need to fit in. Her words highlight the precariousness that she felt as a teacher candidate of color in the process of becoming a teacher. Esperanza may feel that she is an "outsider within" (Collins, 1986) who has gained some level of acceptance within the discourses of mathematics education. Despite

this acceptance, as an "outsider," she may also feel the need to suppress some of the discourses she regularly uses in other areas in order maintain her acceptance within mathematics education, similar to the students of color and women who must change their identity to advance in mathematics (Gutiérrez, 2012b). These selections from her journal will be explored in greater detail in Chapter 7, and they help to explain why Esperanza may have been unable to play in this situation while Karl was. Further this idea was clearly something that Karl had been thinking about and was prepared to ask (he had written about it in his journal prior to this class), while it seems that Esperanza was caught off guard by the question.

Discourses of Constructivism/Recentering Dominant Discourses

Thus far the analysis has primarily focused on Karl's original formulation of the question and Esperanza's response. As the discussion continues in the next section the analysis will shift to look at some of the responses from the rest of the class. There is some further clarification as to what the question was about, and Stella jumps in to shift the discussion explicitly and more generally to the value of abstract mathematics in schools. Notice in this section how Stella and Jeff both construct arguments about what is of most value in mathematics education and how Stella calls her own teaching practices into question. Stella often draws on the discourses of constructivism in mathematics education. Part of the allure of these discourses is that they work against the more traditional (and authoritative) mathematical pedagogies, and, as a result they feel like progress. As a teacher and researcher I encourage my students to use constructivist practices and so when they defend these practices I want to see it as progress. However, constructivism does not necessarily lead to social justice mathematics or more socially

just treatment of students. It can instead, and often is, a means of recentering the discipline, by different means, as it appears to do in this discussion.

119. Stella: Ok so I don't have an answer for that
120. but I was going to rephrase it more generally
121. I was thinking about that question
122. the way that the new core is moving is all about making connections right
123. like it bothers me that maybe Mona would have put
124. solve this system using the equal values method
125. but then when Karl said
126. solve this using completing the square that seemed totally justified to me
127. because it's like of course I want students to make connections
128. and I'll force them to make connections
129. but I don't want to force them to think abstractly[18] right
130. does that make sense so far
131. so so my question is
132. I feel we're all on this like gung ho thing about making connections
133. which I feel like to me is very important
134. but I feel like we neglect some aspect of being abstract
135. like I is it important
136. like why would we even teach our kids the quadratic formula
137. if they could just learn completing the square
138. because it has connections built right into it
139. and they would understand how that connects geometrically
140. but is there a value in learning like the abstract as well?

Lines 141-161 ((discussion about what Stella means by abstract))

162. Jeff: But if you're going to do the geometric case
163. why not tie it to the abstract case?
164. Stella: Yeah.
165. Jeff: I mean is there a reason not to I mean?
166. Stella: Right that's the question sort of
167. Jeff: Karl's question I don't think this stuff helps anybody
168. if there's no context.
169. I mean I that's the problem no connections
170. if you don't make connections

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[18] In this transcript the teacher candidates and I are using the term "abstract" in a different way than how I have been using it in this chapter. In this chapter I use "abstract" to refer to mathematics that is disconnected from real-world contexts and the lives of the students and their communities. While not clearly defined it appears that in this transcript we use "abstract" to distinguish mathematics that does not include a geometric (visual) representation in order to build conceptual understanding. This mathematics would focus more on algebraic manipulation without conceptual understanding. By my definition all of the mathematics discussed in this section would be abstract.

171. there's no there's no structure to hang anything on right
172. Stella: Right
173. Jeff: I mean they need to put this somewhere in their minds
174. so it can make sense right
175. I'm with Karl it makes no sense at all to do that
176. unless they have enough perspective for it to mean something
177. and I don't think you get that at secondary level
178. I mean the quadratic formula first made sense to me in college you know.

In her rephrasing of the question Stella (ll. 119-140) is referencing a discussion from earlier in the class when Karl mentioned asking students to use a completing-the-square method to solve quadratic equations. In this section Stella contrasts this to more "abstract" methods. Stella is talking about a visual, geometric method in which students are literally completing a partially formed square; she considers this less abstract than a purely algebraic approach. While this is still abstract Stella frames it as more relatable to students (because of the visual) since students can "make connections" (l. 132). This emphasis on making connections, which Stella and Jeff mention directly eight times in this section, draws from a constructivist mathematics education discourse. With this discursive shift the focus of the discussion is centered on the mathematical content and no longer on how students experience the teaching or the mathematics. Instead the merits of various mathematical concepts are debated as to their value. Jeff furthers this shift by explicitly linking Karl's question (l. 167) to the importance of making connections. The binary that we appear to be caught in here is that mathematics education is either traditional or constructivist. Thus when Karl questions a traditional practice we automatically try to link it to constructivism as Jane, Stella, and Jeff do, because part of being a "good" mathematics teacher means using constructivist pedagogy. We cannot think of a mathematics education that is not either traditional or constructivist. Thus excludes other possibilities including ethnomathematics and social justice mathematics.

While Stella and Jeff keep students in the conversation the focus on power relations (which Karl introduced) drops out of the argument as Stella unquestioningly and repeatedly suggests that a teacher "force" (ll. 128-129) students to either make connections or learn abstract mathematics

From there Jeff then picks up the discussion (ll. 162-171) and suggests using both (the geometric and the more abstract); in the process he indirectly questions the usefulness of the abstract. By asking "is there a reason not to" (l. 166), he is implying that we should hold onto the abstract unless we have a good reason. While Karl constructed his argument in terms of teacher-student power relations, Jeff (even though he appears to agree with Karl, l. 175) constructed his argument in terms of student mental capacity. He began this part by pointing out the "need" (l. 173) students have to "put this somewhere." But in order for abstract mathematics to make sense students need "perspective" (l. 176), which does not happen at the "secondary level" (l. 177).

Throughout this section students are portrayed as lacking (they do not "have enough" l. 176) and this lack is reinforced later in the class as he points to the "immaturity" of secondary students. Jeff's self-deprecating reference (l. 178) is used to further illustrate the lack of mental capacity of secondary students as a kind of "even I couldn't get it" way that has a different effect than the way Karl playfully mocks his own teaching (l. 6). For Jeff the purpose of this comment is to reinforce his broader point, while for Karl it was to raise a question about mathematics education. In this process Jeff also repositioned the teacher-student relationship such that the teacher is now the only doer ("you" l. 170). This repositioning allowed Jeff to appear to agree with Karl in considering how students experience mathematics education, however, by giving student

mental capacity as the justification, he simultaneously centers mathematics content

Following this negotiation and questioning of the utility of abstract mathematics, Esperanza turned the conversation back to a discussion of the importance of students' "exposure" to abstract mathematics. This time however she made an explicit equity argument and suggested that not exposing students to difficult mathematics equates to not setting high-expectations for them.

179. Esperanza: But to the exposure
180. I was just wondering the exposure
181. that's all I know some students might not even care
182. or quite not understand
183. but just being able to expose them to that kind of mathematics
184. I think it's bringing that equity
185. hello everyone in well in college of course
186. but I mean like you can't consider your students like
187. oh no it's too hard for them
188. we can't give it to them
189. like we were talking to Rebecca about
190. we offer them high level questions or tasks
191. and then we see what happens
192. not easy ones
193. try to practice and do good
194. so that I can give you the hard one
195. and it won't be as hard as it was
196. students are not challenged at all
197. and I mean that's probably the reason a lot of students
198. are not thinking as hard as they should be
199. or you know like thinking constantly as they do math
200. because they've been taught the easy
201. let's use your examples and see how this everything works
202. now you solve it.
203. Teacher: Ok. So I
204. Esperanza: I'm sorry [((unintelligible))
205. Teacher: [No that's good
206. I want to get to some of these points
207. so I'll take two final comments on this
208. and then we'll get into some of these other things
209. I think this conversation has brought up several very important things.
210. Karl: K. So about thinking abstractly
211. like the exposing students to this
212. like I do think it's really important

213. but also like I'm having trouble thinking about where we else we do this
214. like have this more abstract things that are brought in
215. like that's like the quadratic formula is what comes immediately to my mind
216. with a kind of abstract proof that students are exposed to
217. like I don't really I can't maybe can you
218. think of examples like before that or after that
219. because it's like I don't think we do it enough
220. I think our students are kind of blown away when we do it
221. I mean it's probably the only time I've done something like that this year

Here Esperanza makes her equity argument more explicit (l. 184). In particular she emphasizes not leaving abstract mathematics out because teachers might think "it's too hard for them" (l. 187). This may be a response to Jeff's comments about the ability of secondary students to take on the challenges of abstract mathematics. While she never specifies which students she is concerned about, typically conversations around low-expectations have focused on students of color and second language learners. At other times in class Esperanza made it clear that she was particularly concerned with these students in part because of her experiences as a second language learner and a student of color. It seems likely that it is these students that she frames her argument around here. In making this argument Esperanza connects not exposing students to advanced mathematics with low-expectations resulting in a situation where "students are not challenged" (l. 193). As she did previously she frames challenging students with difficult mathematics as a teacher obligation ("you can't" l. 186; "we can't" l. 188), implying that having high-expectations is not a choice teachers should make, but an assumed part of being a teacher. Throughout this portion of her argument, as well as previously, Esperanza replaces repeated emphasis on "exposure" (ll. 71, 85, 179, 180, 183). Here she connects exposure to "high level questions" (l. 190), "hard" questions (l. 194), challenging students (l. 196), and getting students "thinking" (ll. 198-199). This is a

discourse of expectations. Part of being a "good" teacher means having high-expectations of students. High-expectations includes challenging mathematical content, which in school mathematics means abstract mathematics. In this way she links abstract mathematics to high-expectations, thereby solidifying the link between being a "good" mathematics teacher and abstract mathematics. After I make a bid to move the conversation on, Esperanza apologizes, but because I interrupted her the content of the apology was lost. Despite this the apology indicates that Esperanza felt she had broken some classroom rule, and the need to apologize indicates her sense of the precariousness of her position.

After I again state the importance of this conversation (l. 209) and signal that we need to wrap it up (l. 204), Karl addresses Esperanza's point about the importance of "exposure" of students to abstract thinking (ll. 210-212). He both acknowledges and now supports Esperanza's argument for increased access, but in the process he is also less critical of abstract mathematics role in structuring teacher-student power relations. Here he agrees with Esperanza about the importance of exposure and even goes so far as to suggest that teachers "don't . . . do it enough" (l. 216), with the result that students are unprepared to engage with this kind of abstract mathematics. His argument has shifted away from the idea that this kind of practice may be detrimental to power relations. This could be an attempt to merge the competing demands of attention to access to high-level mathematics and power relations. While both arguments could fit within a social justice frame, our discussion has returned to the dominant discourses of mathematics education. The norms of the "good" mathematics teacher have taken our focus off of how the students experience mathematics into a debate on (abstract) mathematical content and

appropriate pedagogies to give students access and help them make connections. The suspension of the typical norms of the class (IRE/F cycles) and Karl's willingness to expose the potential detriment of a moment in his teaching created an opening to disrupt some of these discourses. However, we eventually slipped back into them.

The teacher candidates were clearly confronting a tension between the discourses of mathematics education, which prioritize abstract mathematics, and a desire to improve student access and teacher-student power relations. In the end, the conversation slid back to an emphasis on abstract mathematics and pedagogical practices. Although Karl's question on power relations was somewhat lost in this discussion we returned to this idea throughout the semester to question both their teaching and my own. Karl framed his action research project around this idea and tried to understand how to reduce his use of power in the classroom.

Mathematics Teachers as Clueless Nobility

Another moment of playfulness occurred towards the end of this same class. We first discussed teacher authority (which will be addressed in the next chapter) and the hidden curriculum of school mathematics. Following that conversation we took some time to talk about what students learn about mathematics from the hidden curriculum and where they get these ideas from. This led to a discussion of the historical development of mathematics and of White male dominance in what is now school mathematics. In the transcript that follows Lisa playfully critiques mathematics teachers (including those of us in the class) by comparing mathematics teachers to clueless nobility.

1. Lisa: I think that's true even today
2. they ((students)) say math is not useful in life
3. and we're like

4. “oh you silly like commoners[19]
5. of course you say that
6. you don't think it’s useful” and they're [like
7. Teacher: [“You just don't understand it well enough
8. to make the connection”
9. Lisa: They don't think they're good at math
10. because we reinforce it like
11. “Well give up now.”

Lines 12-20 ((Jeff comments on the importance of mathematics in modern times))

21. Jeff: I mean you went to work in the coal mine or whatever
22. you know if you were a commoner
23. and if you were them ((nobility))
24. you didn't need it ((mathematics)) you know you had money already
25. Teacher: Kind of a game. In some sense.
26. Lisa: Math is a game?
27. Teacher: Yeah. For the elites in that sense.
28. It’s a pretty cool game
29. it’s interesting
30. Jeff: Says the math teacher.
31. Teacher: Yeah. Exactly. And you all should learn it.
32. Jeff: You're so elitist.
33. Multiple: ((laughing))
34. Teacher: So this is this is why I put misconceptions in quotes.
35. Lisa: Yeah.
36. Teacher: Because this isn't how mathematicians think about math
37. but it is how we
38. speaking of math teachers in general
39. have taught students to think about math.
40. We probably weren't trying to teach them to think about math in these ways
41. but that’s the way math has traditionally been taught
42. to emphasize these things ((indicates our list about the hidden curriculum on the board))

As this section begins, Lisa picks up a comment I had made earlier about the influence of European nobility on the development of school mathematics and turns it into a playful (but serious) critique of the role mathematics teachers play in excluding students from mathematics. I describe Lisa’s comment as playful not just because of the laughter it elicits (which it did multiple times), but, more importantly, because of her willingness to

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[19] The use of quotes in this transcript indicates that the speaker is making an imitation rather than reporting speech.

include herself (and the rest of us) in the critique through her consistent use of "we" (ll. 3, 10), which shows her willingness to be a fool. Jeff also picks up on this use of "we" (l. 18) as do I (l. 37). Further the comparison she makes, casting mathematics teachers as clueless nobles and students as disgruntled commoners is not an obvious one. Then Lisa's choice to adopt a mock teacher voice to speak aloud (rather than reporting) the message that mathematics teachers send to students about their mathematical ability (ll. 4-6) adds to this playfulness. This culminates in Lisa's statement to students to "give up now" (l. 11) even though most teachers would never (explicitly) tell a student to give up. Jeff continues Lisa's comparison (ll. 21-24) noting that for the nobility mathematics was not a necessity for subsistence. Responding to Jeff's point I suggest that mathematics could then be a game for them. The nobility did not need mathematics to be practical, even if it was sometimes. Jeff and I play with this idea back and forth (ll. 27-33) in a way that highlights the potential for elitism in approaching mathematics (exclusively) as an abstract game. In this way the discussion has allowed us to reframe mathematics as a game, thus rendering mathematics as less serious than it typically takes itself.

Lisa's initial comment and the discussion that follows occur within the binary that mathematics must either be useful (applied, l. 2) or abstract (pure, ll. 25-26). What Lisa's comparison makes plain is the problematic relationship that this binary sets up between mathematics teachers and students. Within this binary the only possibilities for mathematics are that it be either pure or applied. Pure mathematics is considered the most prestigious and mathematics teachers, as representatives of a kind of mathematics community, feel an obligation to defend it, even when it means putting students down, as Lisa has suggested here. This obligation is connected both to the status that accompanies

pure mathematics, but also because many mathematics teachers enjoy the kind of game that pure mathematics can be. Framing mathematics as a game potentially falls outside the pure/applied binary.

After the laughter subsides, I sum up the discussion by tying it back to where we began (l. 34). I summarize this most recent portion of the discussion (ll. 36-39) emphasizing the critique that Lisa brought in that mathematics teachers are largely responsible for what students think about mathematics. Consistently in our comments Lisa, Jeff, and I have placed responsibility on mathematics teachers for the ideas that students develop about mathematics. However, as I continue I soften this critique by excusing mathematics teachers from this responsibility (ll. 40-42). I am in these lines trying to be a "good" teacher educator. As a "good" teacher I do not want to place blame on the mathematics teachers these teacher candidates work with nor on the teacher candidates themselves. I had been concerned going into this class that the teacher candidates would be resistant to some of my critiques of mathematics education. In order to reduce (potential) resistance I softened our critique by pointing out the lack of intentionality (l. 40) on the part of mathematics teachers. Connecting intentionality to responsibility, as I did here, is a classic way to maintain a White sense of goodness (Applebaum, 2010), and deflect taking responsibility for making meaningful change. I am attempting to make this class a "safe space" for the teacher candidates, but also for myself, as a White teacher. However, when making a conversation safe becomes a means of avoiding potentially difficult conversations then the dominant (White) discourses are maintained (Yoon, 2012). By bringing normative goodness back into the discussion I weaken Lisa's critique and reinforce the traditional teaching of mathematics that excludes

so many students.

Goodness Means Following the Rules

From here Jeff picks up the discussion. From this momentary disruption he appears to be trying to find "the way" (l. 64) to teach instead. He seems to accept (or at least consider) that what has been done is not working and is wondering what to do that would be better. As he does he points to the difficulty he feels in trying to change from a traditional way of teaching. As he notes, this is a "struggle" (l. 57) for him, because as he says, "I don't understand quite what the goal is" (l. 67). However, the fact that he is even bringing up the issue shows a shift in his thinking about mathematics education. Note how he now uses the word "abstract" to refer to nontraditional mathematics teaching (l. 69).

57. Jeff: I mean but I mean I really struggle with this concept a little bit
58. because there isn't any question that we all experienced
59. I mean we're here because mathematics as it has traditionally been taught
60. worked for us.
61. Teacher: right
62. Jeff: If there's a different way of doing it for our students
63. you know which is different from we
64. I don't know we've necessarily hit on the way
65. we're looking for it perhaps
66. but what we don't
67. I don't understand quite what the goal is for us
68. or what it ought to be for them
69. we always we have these abstract ideas
70. like we want them to think better for themselves right
71. that doesn't really mean anything to them you know
72. and so what's our goal
73. what do we want them to walk out of our classes being able to do
74. that's measureable
75. so we actually knew we did something
76. I I'm confused about some of this
77. I know what it means to me
78. and I think it's similar to what most us in terms of our experiences
79. but I don't know what it's going to look like to a kid who never

80. I mean people are different right
81. I mean what are we expecting them to come out of our classes with?
82. Teacher: So if I can phrase your question in broader terms.
83. What is the purpose of math education?
84. Jeff: Right probably yeah I mean this beyond
85. yeah they need to get a job,
86. you know beyond that.
87. Teacher: Um I don't know that there is a single answer to that question
88. but I think it's a very important question to think about
89. as math teachers
Lines 90-128 ((I comment on how mathematics has served to divide students into tracks and we briefly discuss job preparation))

After noting his own struggle with the idea of not teaching mathematics in a traditional way he brings up the possibility of teaching in "a different way" (l. 62), but adds that he does not know what that way is. While he is struggling with this concept, his question assumes that there is a need for a different way. Part of the struggle for Jeff is that teaching in new ways is "abstract" (l. 69) for him. In other words, for Jeff and all of us, this different way of teaching lies outside what is thinkable within the dominant discourses of mathematics education. Jeff wants this different way to be more concrete and familiar as he indicates by drawing on the discourses of standards and standardized testing, since these are perceived to define what can and should be taught. He is looking for a "goal . . . that's measurable" (ll. 72-74) and is focused on student outcomes ("walk out . . . being able to do" l. 73; "come out of our classes with" l. 81). This language reflects a traditional metaphor of knowledge, which is transmitted from teacher to student and is something that students can pick up and take with them, rather than something that is socially-constructed by teachers and students in the moment of learning. While this thinking may address Gutiérrez's (2012c) dominant axis (focusing on access and achievement), a different conception of teaching and learning is required to connect with identity and to confront power relations.

In a sense Jeff is looking for a set of rules to tell him what to do instead of the traditional ways of teaching. He is uncomfortable with the "abstract" (l. 69), the lack of a clear goal (ll. 67, 72), and the confusion (l. 76) that he feels as he considers moving away from the certainty of traditional ways of teaching mathematics. This feeling is not exclusive to Jeff and is likely common for any group of teachers (especially teacher candidates) trying to take the uncertain path trying to break out of dominant discourses. However, for a mathematics teacher to take on the equity stance proposed by Gutiérrez (2009) requires that teachers embrace the uncertainty and tensions in teaching mathematics for social justice. The just-tell-me-how-to-do-it approach oversimplifies what inherently is a complex, life-long process in a way that makes teaching for social justice more comfortable for White teachers (DiAngelo & Sensoy, 2010), because we can avoid the critical evaluation of our own motivations and practices (Applebaum, 2010). I do not want to give Jeff a direct answer or provide him with rules for how to teach mathematics nontraditionally and so I rephrase the question more broadly to focus on the purpose of mathematics education (l. 83). Jeff recognizes that the purpose of mathematics education needs to go beyond the purpose of education within human capital ideology "to get a job" (ll. 85-86). The question of the purpose of mathematics education is important. It is not one that we took time to consider in this moment or in depth at other times in the class. In the future I plan to make this consideration an organizing theme of my classes.

Access as Opening Gate/Shifting Fences

A short time after Jeff's comments about the challenges of teaching nontraditionally Stella returns to the idea of access. As she does so Stella uses two metaphors that highlight the discursive trap that can accompany an access only approach

to social justice. In the question that she poses to her classmates Stella connects access to and exclusion from college to abstract mathematics.

160. Teacher: Go ahead.
161. Stella: I was just going to remark
162. that this is sort of connected to what my question was before
163. there's two schools of thought
164. I learned at our conference.
165. There's like a school of thought
166. that's like provide everyone access
167. so that like people can overcome these boundaries of privilege
168. and then there's a school of thought
169. that's like no that will never happen
170. because you are just shifting
171. what the fence is you know
172. Jeff: You're raising the average overall.
173. Stella: Yeah. Once people know math
174. there's just going to be another gap of inequality somewhere else
175. and so you're really just shifting things around
176. so that was my question about like the purpose of the quadratic formula
177. is that just for students to like know a
178. to be exposed to some sort of like that kind of abstract
179. or symbolic thinking
180. so that they can be successful in college
181. so that they can have access
182. is that what that's for
183. or is the nature of that important in itself
184. like is it good for students to be exposed to that for some other reason?
185. Does that make sense?
186. Jeff: If you need to find the zeros for some reason [then
187. Stella: [But will you ever
188. Jeff: Well that's I was thinking
189. that very question how many of our students will leave our math classes
190. and go out into the world
191. and think I need to find the zeros of this.
192. Teacher: How many of you
193. let me just make a comment
194. how many of you um in your life outside of school
195. have used something calculus or higher to solve a problem that you were facing?
196. I can think of one time in my life.
197. Jeff: I remember doing something
198. but I can't remember what it was to be honest.
199. Teacher: And I don't think I could do it now actually
200. I don't remember calculus well enough to do it um
201. Esperanza: What did you have to do?

Lines 196-215 ((I give an example of a time I used calculus in a job))
216. Lisa: Sweet.
217. Teacher: One. One time. Ok, Gavin what was your comment?

As Stella begins her point, she draws on the language of equity to focus on "access" (l. 166). Stella then uses two metaphors to critique both the access argument that Esperanza (note her use of "exposed" ll. 178, 184) made earlier as well as some of my earlier comments about mathematics as gatekeeper. In the first metaphor she characterizes mathematics (the connection to mathematics is made explicit later, l. 173) as a set of "boundaries of privilege" (l. 167). This boundary metaphor is consistent with the dominant liberal view (seemingly the perspective that Esperanza and I were working from) that a teacher merely has to open the gate to "provide everyone access" (l. 166) and then students "can overcome these boundaries of privilege" (l. 167), seemingly on their own. This perspective ignores the ways in which mathematics-as-discourse actively excludes students of color and women. Stella also includes an agent (the mathematics teacher) who can "provide everyone access" (l. 166) who, in this way, may operate as a lone hero who will swoop in to save these students (Thompson, 2008). In contrast the second metaphor is that of a "shifting . . . fence" (ll. 170-171). In using this metaphor Stella suggests that as mathematics teachers open gates to more students the rules (or fences) will change to continue to exclude many students (l. 174). While Stella suggests that the fence would change from mathematics to something else ("another gap" l. 174), the metaphor could also be used to suggest that the fence would change from one level of mathematics (say Algebra II) to another. Again, in this metaphor, Stella points to an agent ("you" ll. 170, 175) who may think that she/he is providing access, but ends up "just shifting things around" (l. 175), with the end result that certain students are still

excluded, just by different means.

The contrast of these metaphors then sets up the framing of her question around the "purpose of the quadratic formula" (l. 176). Her question exposes some of the binary thinking that occurs within mathematics education (and in our own thinking in this class) where either there is something about the mathematics that is "important in itself" (l. 183,) or just for "access" (l. 181,). She has just proposed that the access argument is not very compelling and Jeff (ll. 186, 188-191), Stella (l. 187), and I (ll. 192-196) each point out the lack of utility for the quadratic formula specifically and abstract mathematics generally. Mathematics teachers' responses to the question "When will we ever use this?" often reflect this same binary. If the teacher knows an application, often obscure or in a scientific field, she will share it. Otherwise the answer is often something about end of year tests, college, or future mathematics classes (access). We are also stuck in this binary, at least for the moment, because these are the options presented by the discourses of mathematics education within the context of abstract mathematics. We have critiqued both parts of this binary, but do not yet have a good way to break out of it. These metaphors made plain the way this binary restricts us. However, by not following up we missed an opportunity to consider alternatives and to explore what equity means beyond access. While this may include achievement, identity, and power, classes may explore other possibilities.

Disrupting Social Justice Mathematics

Towards the end of the semester Karl taught one of his classes a social justice themed lesson. He explained that he had modified a social-justice lesson developed by Lisa, Stella, Esperanza, and Jane. This lesson focused on the amount of park space in

lower- and higher-income neighborhoods in our city, with the clear result that families in lower-income neighborhoods had less access to parks. This was one of the only instances where a teacher candidate talked about having tried a social-justice lesson. The class that Karl taught this lesson in was the smaller of his two classes. Most of the students in this class were ELL students, and many were also refugee students, and had a history of low grades in previous mathematics classes. As Karl explains the lesson did not go as he had planned, but it still challenged the standard relationships in his class.

1. Karl: um in on a side note
2. but related I guess in terms of authority
3. we did uh your guys' park thing
4. so I kind of was going to give a brief thing on that

Lines 5-14 ((Karl explains how he adjusted the worksheet and the mathematical focus of it))

15. so like that was kind of the math reason for doing it
16. but the discussion was pretty good
17. it started out kind of poorly I thought
18. just because maybe I didn't I didn't expect the response initially to be like well
19. to start talking about this
20. and students would be like basically say I don't give a s____ about parks
21. like why are we doing this
22. I don't care like to be kind of like
23. well what would be fair
24. and they would say let's just have it how it is now
25. I don't care I don't care what parks are like
26. and that was about the first five minutes of the discussion was about
27. how people don't really care about parks
28. and I had like a little bit of a discussion a little bit of like
29. uh talk like a little bit more serious like
30. hey like this I know we're talking about parks right now
31. but take this a little more seriously for a little while
32. this isn't really about parks
33. and I think you guys will figure out what's going on here
34. and we talked about that a bit and the conversation got really good
35. and some things that I mentioned to
36. I think the two of you yesterday
37. that they brought up were things
38. that I never thought about in terms of this park uh issue
39. and one thing being cost
40. or a couple things being cost of upkeep for a park

41. I didn't really think of that for as a possible reason
42. for maybe how they're distributed
43. like maybe one park has more graffiti than another park
44. and more cost is gone into upkeeping it
45. one student brought up the distinction between a community park and a city park
46. and I didn't know the difference really
47. and she was kind of thinking she was like schools are kind of supported
48. and I don't know much about taxes
49. and she was kind of asking me about taxes
50. and I was like I don't know your idea could be right
51. or it could be wrong
52. so she was kind of wondering if taxes from a certain community
53. kind of go into parks
54. or if it's all everything from the city goes into one big fund
55. and then it gets distributed into all of the parks
56. or for instance there are some parks in Salt Lake that are funded privately
57. or from say the Sugarhouse community
58. which I thought was an interesting question
59. that I didn't expect from a 17 year old
60. but I think in terms of authority I think it was a really good thing to talk about that
61. and we did even bring up that from there
62. I kind of talked about that maybe you mentioned someone mentioned that the
63. um the way to get in state tuition in Utah as opposed to like other states
64. and so we brought that up
65. and we had a short discussion on that and
66. students were really interested in that
67. see this is kind of like more what this parks discussion is about
68. like this is an opportunity in Utah
69. this is these other states keep thinking about this
70. so just wanted to talk about that as an aside

Karl evaluates the overall discussion as "pretty good" (l. 16) even though it started "poorly" (l. 17). From the dominant view of social justice mathematics lessons, which are supposed to engage students in social critique through mathematics, this lesson could easily be described as unsuccessful and Karl may have also initially responded from this perspective. However, Karl frames this anecdote as being about "authority" (l. 2, l. 60), not just about social critique. This framing provides a more nuanced view of the lesson and suggests possibilities for a broader view of teaching mathematics for social justice. He notes that he did not expect the negative response (ll. 20-22) and this may be a

common experience for a mathematics teacher trying out a social justice lesson, especially when the idea for the lesson did not come from the students but from a White teacher. After his students expressed their lack of interest in the lesson (ll. 25-27) Karl convinced them to give the lesson a further chance and to consider things other than parks (ll. 30-33).

Here he notes that then the "conversation got really good" (l. 34); this happened in part because his students participated in unexpected ways (ll. 37-38). His students' ideas included the cost of upkeep for a park (l. 40) and how parks are funded (ll. 45-49). He repeatedly points out how these were ideas that either he had not thought of (ll. 38, 41) or that he did not know (ll. 46, 48, 50). This expresses a kind of surprise (see also "I didn't expect", l. 59) at the level of thinking of his students and suggests a nontraditional classroom interaction that temporarily shifts the power relations in the class. While occasionally expressing not knowing may be common, doing so this often is rare. The kind of interaction described here in which students are bringing up unexpected ideas and asking questions that the teacher does not know the answer to are not the kinds of questions that, usually, come up in a traditional, abstract mathematics lesson. This suggests at least for the time of this lesson a more equal relationship between teacher and students. Even though students may not have been interested in parks, engaging in the discussion resulted in a different dynamic in the relationship between teacher and students. This discussion also led Karl to bring up the (apparently) unrelated topic of in-state tuition for undocumented immigrant students (l. 63). It appears that broaching the topic of one form of inequity opened up a discussion that allowed Karl to also address other forms of inequity.

This brief example is important in that it shows the possibility of multiple ways to address social justice in mathematics education besides the social critique lessons that are becoming the norm for social justice. Even though this shift in power relations occurred when addressing a social justice topic it suggests a possibility for addressing social justice on a more regular basis (social critique lessons are usually a small part of a curriculum, Gutstein, 2006). It also suggests possibilities for mathematics teachers who are currently unable (due to their own skill or teaching context) to engage in social critique lessons. Significantly examples of multiple ways to approach social justice mathematics can help to avoid one version becoming dominant.

Dominance of Abstract Mathematical Discourses

Week 1: Mathematics Is Abstract

While the moments when dominant discourses are disrupted or made plain are instructive, it is also important to understand the moments when this did not happen. In the following section of this chapter I turn to our class discussions where we remain solidly within the dominant discourses of mathematics education. These discourses are centered around the idea of mathematics as naturally and uniformly abstract. Abstraction is one way that mathematics serves to maintain White privilege by appearing neutral, while creating a racial hierarchy. The discussion that follows, from early in our first class, illustrates the dominance of the discourse of mathematics as abstract. This discussion occurred as I was giving the teacher candidates an opportunity to get to know me better. To this point my relationship with the teacher candidates had been limited primarily to supervision of their student-teaching. As class began I provided a time for students to ask questions about me that were not about the class. I wanted them to get to know me as a

person and not just as their teacher. This was an intentional move to reverse the standard teaching emphasis on getting to know the students, which views the teacher-student relationship as hierarchical rather than reciprocal.

Among the questions that they asked me was one about what research I was doing for my dissertation. I was not well prepared to answer this question for my students and was unsure how much of my viewpoint (on social justice and abstract mathematics) I should reveal to them at this point. Most of them knew of my interest in social justice mathematics (I had given a guest lecture in their multicultural education class), but we had not discussed at length what social justice means. I decided that being open with them, without going into lengthy theoretical explanations, would be the most beneficial in establishing the relationships we would need in this class.

Lines 1-20 ((I summarize my research and questions for my students))
21. Jeff: So are you talking about the teaching aspects of ((unintelligible))?
22. I mean at some point I don't know what it means.
23. I mean mathematics is abstract at some level I mean.
24. Teacher: It is because that's how ((Jeff tries to break in here but I talk over him))
25. because that's how people have made it.
26. Jeff: But so in other words you think there are structures that might exist
27. that make it less so, is that [
28. Teacher: [Yea. For example uhm
29. Jeff: And if this isn't constructive I'm happy to stop.
30. Teacher: No it's a good question . . .

As I introduce my research I bring up the idea that mathematics as abstract is not the only possible form of mathematics. In response, Jeff states (l.20), that "mathematics is abstract." This assertion is not surprising. It is unlikely that Jeff or any of the other teacher candidates have had experience working with mathematics that is not abstract (or, if they have, it has not been called mathematics). Thus the simple naming of what is mathematics divides our world into things that are mathematics and things that are not.

As mathematics is constructed in dominant discourse, only those things that are abstract can be considered mathematics. Stating this in such a straightforward way, without modifiers and using "is" presents mathematics as naturally abstract. The dominance of this discourse limits our ability to imagine other forms of mathematics (that are not abstract) and thus our ability to teach in other ways. In this statement there is no room for other forms of mathematics since abstractness is presented as one of the defining characteristics of mathematics.

In my response to Jeff, I point out that people made mathematics (l. 25) by noting that mathematics is abstract only because people have made it abstract. In this way I challenge the naturalized view of mathematics as abstract to emphasize its social construction. However, at this point these teacher candidates may not be familiar with the language of normalization and likely have not heard of (or not recognized) mathematics when it is not abstract. In an attempt to help them consider the possibility of other potential forms of mathematics I explain an example given by Rose (2012), and which I reference in Chapter 2. In this example Rose explains the mathematical concepts involved in a welding class. While the welding instructor does not seem to doubt his ability to teach welding, he does not see himself as capable of teaching the mathematics. This occurs in part because generally mathematics is only recognized by the symbols and textbooks that we associate with abstract mathematics. Notice here that while Jeff continues to emphasize the abstractness of mathematics he, along with Gavin, begins to allow for other possibilities, even though, at this point, these possibilities are limited.

Lines 31-52 ((I give an extended example of the mathematics in a welding class from Rose, 2012)).
52. Teacher: They probably don't see it as mathematics
53. and the instructor in this example also didn't see it as mathematics

54. because of the way we have developed mathematics
55. and the people who developed mathematics
56. and the way they defined it uhm had mathematics developed differently
57. that kind of very practical application could be what mathematics was now,
58. but that's not how it developed
59. Jeff: Ok. I'm a skeptic, but I'm [
60. Teacher: [That's ok.
61. Jeff: I mean I don't see how you are going to do some things
62. without being abstract
63. but I don't see . . . there usually is an application of this not abstract or as abstract
64. so I'll go with it for a while.
65. Teacher: Ok.
66. Gavin: I agree with you
67. and the more I've thought about it like when you go into modeling life situations
68. that's when it becomes less abstract and more applicable
69. Jeff: But the question is can you develop it
70. actual mathematics as a more hands on.
71. I think that's kind of what you're saying.
72. Teacher: Yea.
73. Lisa: Well and then isn't that just like the definition of mathematics?
74. Teacher: Right. The way
75. Lisa: If you're not talking about mathematics the way
76. that you're used to,
77. it could be anything.
78. Teacher: I'm talking about a broader idea of what mathematics is.
79. Jeff: I will endeavor to be open minded.

As I close the welding example I emphasize (through repetition of "developed" ll. 54-58) that mathematics is a human creation. In response, while Jeff announces his skepticism (l. 59) he also positions himself as open to the possibility that there "is an application" (l. 63) of mathematics that is not as abstract. Gavin also supports this view as he mentions "modeling" (l. 67). While they are holding firmly to the idea of mathematics as abstract they are also signaling a willingness to consider other options. However, both have also positioned themselves as judges of what will count as mathematics or not. Importantly, both are still operating within the binary of pure (abstract) and applied mathematics, in which applied mathematics is understood as taking some aspect of pure mathematics and using it to understand some applied problem. If the welding example I give is viewed

from within this binary it is not applied mathematics until someone takes pure mathematical concepts in order to understand what is happening. From this perspective the welding example is not mathematical by itself until it is described mathematically.

Neither Jeff nor Gavin is talking about the idea of application in the same way that I am; they are both taking an abstract idea to an application where I am suggesting that mathematics be developed from applied contexts. However, by using the verbs "develop" (ll. 54-55) and "broaden" I imply a direction or progress to that development as well as broadening out from some center. The likely assumption is that it should develop towards the abstract or broaden out from the dominant center. Thus even as I challenge abstract mathematics I also maintain its position as dominant, because I am operating from within the same binary, even though I am trying to reverse it.

Lisa's comment at the end that "it [mathematics] could be anything" (ll. 75-76) also shows her willingness to question abstract mathematics and to participate in the class. Her statement also goes well beyond those of Jeff and Gavin. Whereas they allowed for a narrow range of possibilities, which they would evaluate, Lisa has opened it up to "anything." Lisa's statement does present a way out of the pure/applied mathematics binary that Jeff, Gavin, and I were all using so far in the discussion. However, this wide range may inadvertently serve to close off possibilities for further discussion since it basically makes the argument that definitions are arbitrary, making everything relative. These arguments about arbitrariness can make discussions slippery; they make it difficult to maintain a disruption of abstract mathematics since everything depends on definitions and assumes that we have the power to redefine mathematics in any way that we choose. We do not have this power as we are confined to work within

the existing discourses on the nature of mathematics and an explanation of power and dominance would need to come into the discussion at some point. From here the discussion moved on to respond to other questions, the nature of the course, and defining action research. However, abstract mathematics would be a recurring topic throughout the semester.

Abstract Mathematics and the Exclusion of Social Justice Mathematics

This understanding of mathematics as naturally abstract, and our own affinity for abstract mathematics, limited our understanding of teaching mathematics for social justice. One of the challenges in preparing mathematics teachers to teach mathematics for social justice is that mathematics teachers generally do not have any experience (as students or as teachers) with this kind of teaching. This was part of Jeff's struggle in asking for clear rules and definitions about what it means to teach mathematics for social justice. Further the dominant discourses of mathematics education typically extend only as far as a limited understanding of access (occasionally achievement) making discussion of what alternatives to dominant mathematics may look like, difficult. We encountered this issue during our fourth class when we got stuck in the binary of trying to make abstract mathematics more practical and/or giving more students access to abstract mathematics.

Part of my plan for these teacher candidates was to engage them in some of the theory behind social justice mathematics and then work with them in some practical discussions of the implications of that theory to their teaching. In general a portion of each class was devoted to a discussion of how the teacher candidates could apply that

day's topic to their own teaching. The most clear attempts to do this in regard to social justice included the Creating Balance in an Unjust World: Conference on Mathematics Education and Social Justice that I attended with the four women in the class during the second week of the semester, two classes (week 6 and week 12) focused on readings about social justice in mathematics education, and smaller activities embedded in other classes. After attending the conference the four women shared some of their experiences with the three men in the class. In general the teacher candidates were very interested in the readings and readily engaged in the discussions. However, when it came time to discuss the relevance of the readings to their teaching we found it challenging, in particular because of the discourses of abstract mathematics.

Abstract Mathematics "Works"

To prepare for our week 6 class the teacher candidates read articles by Secada (1989), Gutiérrez (2012c), and Stinson and Wager (2012). We spent the majority of this class discussing the ideas presented in these articles. Towards the end of this class we begin a discussion on ethnomathematics. Work in ethnomathematics provides possibly the most significant challenge to the dominance of abstract mathematics by redefining and reframing what mathematics can mean. The preservice teachers had read an article by Gutiérrez (2012c) in which she briefly mentions the work of D'Ambrosio and ethnomathematics as a means to connect better with students' mathematical knowledge and identities and to critique power. We had already discussed other aspects of this article including Gutiérrez's equity framework including access, achievement, identity, and power. My question about alternative notions of knowledge led to the discussion transcribed below.

1. Teacher: Um alternative notions of knowledge, what does that mean?
2. Lisa: isn't this the ethnomathematics guy?
3. Teacher: Yes D'Ambrosio is the ethnomathematics guy.
4. So what does that mean?
5. Lisa: Well what I think it means is
6. sort of like what you're studying alternative notions of knowledge
7. to me that means
8. like ok we think of math as a certain thing
9. but there are alternative notions of what math is
10. Gavin: I get this perspective, right? Math is taught in such a way
11. that it has a single perspective usually
12. but you can approach things from different ways
13. to come about it the same.
14. Jeff: Well that's kind of what I was trying to understand
15. when that whole
16. when you actually
17. I was going to ask you about that
18. what you mean
19. because I mean when you start talking about the fundamentals of mathematics
20. mathematical logic
21. you know axioms
22. I mean are you saying
23. that there's that we should just rearrange our assumptions
24. what is it you're actually saying
25. is it more from how we look at it from a teaching standpoint
26. is that what you mean?
27. Teacher: Yeah I think we need to broaden our idea of what math is
28. and invite and welcome in alternative perspectives
29. now you still have to teach the mainstream math
30. because they are going to need to know that for future classes
31. and for tests and things
32. but if if they can use alternative ways of understanding math to access
33. Jeff: So but that's important for me to
34. so when you say alternative ways of understanding math
35. that to me is a different that's a different sentence
36. than alternative ways of know
37. because when I think of knowing
38. I think well the basis for math
39. I mean it does all these
40. I mean the reason we use it is because it works
41. I mean if we're going to change it ((laughs))
42. I was trying to struggling with
43. Teacher: But it's not the only way that works

After I reassert the original question about what "alternative notions of knowledge"

means, Lisa and Gavin both give somewhat vague answers that make connections to ideas that I have mentioned before or that they have heard in other classes. Lisa's comment is mainly an assertion that "there are alternative notions" (line 9) without explaining what they might be. Gavin tries to link this idea to "approach things from different ways" which seems to be solving the same problem in different ways, but in the end getting to the "same" (line 13). Jeff continues this vague line of thinking and answers my question with his own question while making reference to "mathematical logic" and "axioms" (ll. 20-21). This area is one of his strong points and an area where he may feel that he has stronger understanding than the rest of us do. In a sense (whether intentional or not) this is a way to assert his mathematical authority. The lack of clear answers at this point is understandable as this class is likely one of the first times they have considered alternative forms of mathematics and we have not spent time studying this idea to this point.

Beginning in line 27 I respond to their statements. In this response I mostly position "mainstream mathematics" as still more important and dominant. While I am attempting to challenge the authority and dominance of mainstream mathematics I am also reasserting its dominance. I do this by using the terms "broaden" (l. 27), "invite" (l. 28), and "welcome" (l. 29). The term broaden suggests expanding out from some core (such as dominant mathematics) and invite and welcome imply that we (as mathematics teachers) are discursively located with dominant mathematics and we bring in other perspectives, but that these perspectives are outside or foreign. Then in line 32 I situate alternative forms of mathematics as a way to provide "access" to school mathematics. Thus I am positioning alternative understandings of mathematics as only useful if and

when they connect to the mainstream. The teacher candidates' initial attempts to consider alternative forms of knowledge and my response to them illustrate the dominance of abstract mathematics, in that we are unable to think of mathematics that is not abstract, and even as I try to challenge it I am unable to do so in a way that does not simultaneously recenter abstract mathematics.

At this point Jeff jumps in, in order to clarify what I am saying. After multiple restarts he asserts his main point, which is that "we use it [abstract mathematics] because it works" (l. 40). In this way Jeff positions abstract mathematics not as a human invention, but as inevitable and natural; he is not questioning the dominant position of abstract mathematics because there are no alternatives that are meaningfully different, at least not that we can think of. After suggesting the possibility of "change" to mainstream mathematics he briefly laughs, possibly suggesting that the idea is ridiculous to him. A conversation that is supposed to be in support of alternate forms of mathematics has quickly slipped into a recentering of mainstream, abstract mathematics.

As our discussion continues, note how abstract mathematics is still centered even as we provide alternative examples.

43. Teacher: But it's not the only way that works
44. Jeff: And if there is another way then that's fine
45. I don't know what it
46. and someone would have to determine what that is
47. um anyway I mean I'm saying calculus is calculus
48. the fundamentals of calculus
49. is there another way to think about calculus
50. well I don't know
51. Teacher: Certainly there is Leibniz had one way and Newton had another way.
52. Jeff: Well ok.
53. Gavin: but
54. Teacher: They were fairly different. One became dominant
55. Jeff: Ok. The notation according to Leibniz became dominant.
56. Teacher: It was beyond notation though

57. but that's one example.
58. do you want to give your boatmen example?
59. Stella: Ok. So these people I was obsessed with
60. gave this
61. the lady ((Swapna Mukhopadhyay)) had done like fieldwork in India
62. with boat making men
63. and um she's they're like illiterate basically
64. but they're known for making these beautifully famous boats
65. and they're parallel to boats that really educated engineers here would build
66. but they don't write anything down
67. while they are building them
68. they don't know how to read or write
69. and they can still like can come out with the same product
70. and she was talking about how they know how much force to apply
71. when they're bending the wood in a certain way
72. they just they say that they can like feel it in their body
73. so they
74. it was like a bodily experience with math
75. I don't know
76. also I thought it was interesting that
77. they they don't even have the word symmetry in their language
78. but they like know it really deeply
79. they build them perfectly symmetrically
80. and so I don't know
81. it's just this idea
82. that the way you communicate math ideas
83. doesn't dictate how valid they are
84. you know I don't know.

When I suggest that there are other possibilities (l. 43), Jeff effectively dismisses this with "that's fine" (l. 44) and "I don't know" (l. 45) and then "anyway" (l. 47) to bring him back to the real point, which he reasserts saying "calculus is calculus" (l. 48) again pointing to the unquestioned dominance of mainstream mathematics. His comment that "someone would have to determine what that is" (l. 46) suggests that considering these questions (about the nature of mathematics) is outside the domain of a mathematics teacher. This idea of determination suggests an authority of judgment of what is and is not mathematics. We go back and forth a bit as I propose an example that Jeff is unconvinced by and then I call on Stella to share an example that she has shared with me

in the past.

In Stella's example she sets up a contrast between the "illiterate" (l. 63) men and "really educated engineers" (l. 65) such that both "can come out with the same product" (l. 69). In this way she is also making the mainstream (at least for us) the standard of comparison, rather than evaluating the Indian men's work on its own merits. She gives the example in a straightforward, confident manner (note the lack of hesitation and lack of qualifying statements); however, towards the end when she gets to the purpose of the example (indicated by "so they" l. 73) she becomes more hesitant and unsure, as noted by the regular use of "I don't know" (ll. 75, 80, 84). It is at this point that she stops reporting an example that she has heard (from a mathematical expert) and begins making an interpretation of it. In this discussion her interpretation is the first potential example of an alternative way of knowing, which she describes as a "bodily experience with mathematics" (l. 74) and "know[ing] it really deeply" (l. 78). Implicitly these deep, bodily ways of knowing are contrasted to the primarily mental and superficial ways that students often experience with abstract mathematics.

Stella's final summary statement is important (ll. 81-83). Here she introduces the idea of communication. Communication has not been mentioned to this point, although during other class meetings the teacher candidates discuss the value of abstract mathematics in facilitating communication of ideas. Stella makes an attempt to break the link between method of communication (where symbols and form are dominant) and the validity of the ideas. Even though there is nothing in her example about communication, she is suggesting that the reason that an engineer's knowledge is valued more than an Indian boatmaker's knowledge is that the engineer communicates in the accepted way.

This idea of communication is part of the discourse of the universality of abstract mathematics. Part of what is considered to be an advantage of abstract mathematics is that it is perceived as universal, and as a result language-neutral. Thus ESL students are often expected to do well in mathematics classes, because mathematics is considered a universal language. This point about communication will keep coming up as the discussion continues.

So far all of us are struggling to understand alternative ideas of mathematics in a way that is not subordinate to mainstream, abstract mathematics. Jeff's struggle is more explicit and doubtful for the moment, Gavin's and Lisa's ideas are vague, and while Stella and I provide alternative examples, we also reassert the dominance of mainstream mathematics, although Stella has attempted to separate the quality of an idea from the means of communication. While Jeff challenges the ideas we are discussing, I do not believe this is active resistance on his part; instead it comes from a genuine desire to understand how to apply these ideas to his teaching and a struggle to fit these ideas with his understanding of what it means to do and teach mathematics.

Alternative Forms of Knowledge in a Lesson

As we continue Jeff asks about the applicability of Stella's example to teaching. Stella responds with a plan she, Jane, and Lisa have developed to engage students in a lesson that challenges dominant discourses of mathematics. However, I and the rest of the class miss the implications of her suggestion.

85. Jeff: I'm happy with those kinds of examples.
86. Can you teach that to like large
87. I don't know
88. I'm just it not that
89. ((overlapping talk))

90. Esperanza: Can you transfer that you mean?
91. Jeff: Yeah how do you transfer that?
92. Gavin: I think it is just that it is an example
93. so when people are like saying
94. where does that apply
95. people do this naturally
96. and something I read this week about in Cangelosi's book about inductive reasoning
97. it's about dealing with specifics to get to the point of the abstract right
98. and so that's an example of a specific
99. so the boatmakers are dealing with a specific in mathematics
100. and they may or may not generalization
101. but there is a generalization that can come through that specific
102. and students learn through specifics to get to the abstract you know
103. if necessarily like teaching them how to like feel this bodily experience
104. but like using it as an example
105. that math does exist outside of this abstract notion
106. that we have within this class
107. is something that's valid.
108. Stella: Yeah and um
109. so we're actually going to do this in our lesson study in Marin's class
110. but we're going to like tell that story
111. and another part of the story that I left out
112. is that they don't have any tools of measurement
113. they'll use like a rope that they use for other purposes
114. or their saw or something to measure so
115. anyway we're going to do
116. we're going to like tell that story
117. and do a project
118. where they're measuring perimeter or something using different
119. a perimeter of this square using different objects
120. like straws or paper clips or whatever
121. and then also using standard units of measurement so
122. and we're going to ask them like
123. do you think if you're going to build a boat
124. what are you going to use to measure with you know
125. and then we're going to talk about you know
126. is it any more valid of a measurement
127. just because it's a standard units of measurement
128. I don't know
129. I feel like just bringing up those ideas that
130. that's not the only way to do things
131. or learn math
132. or participate in math
133. just the way that other people have done it
134. you know what I mean.

Jeff again challenges the utility of Stella's example for teaching and instead of me responding to his question, Gavin and Stella step in. In his response Gavin attempts to fit the example of the Indian boatmen into the pure/applied mathematical binary. He does this in a slightly pejorative way ("just . . . an example" l. 92) and links it to the applied (lower status) side of this binary (l. 94). Gavin then connects Stella's example to the idea of "inductive reasoning" (l. 96) that he is learning about in another class. In the process he positions Stella's example as a "specific" (ll. 97-99) that can help get students to the "abstract" (l. 97) and to "generalization" (ll. 100-101). So, as I also did, Gavin recognizes the example as "valid" (l. 107), but only because it may eventually connect to the "abstract."

Stella follows up on Gavin's comment (beginning l. 108) and continues her previous example. This time though she applies this to her own teaching. She is able to do this more successfully than the rest of us perhaps in part because she has had more time to think about this and has worked with others (including Jane and Lisa) to prepare a lesson for one of her other classes. Beginning in line 122 Stella explains that she and her classmates will explicitly bring up the idea of the relative validity of standard and naturalistic tools of measurement, in order to challenge the idea that the standard is the "only way to do things" (l. 129). In this way she is able to apply the idea of alternative understandings of mathematics to the context of a specific mathematics lesson on measurement and perimeter. This is significant because it illustrates how she successfully inserts some aspects of social justice mathematics into a mathematics lesson. Specifically Stella is doing work to undermine the way a standard unit of measure (as a dominant discourse) serves to exclude alternative ways of measuring. This work can potentially

link to Gutiérrez's (2012c) critical axis, because it creates discursive space for students to use their own methods of measurement (attending to identity) making the dominant discourses more visible. It seems that she was able to do this because she has spent more time thinking about this idea (about 3 weeks from when she first heard this experience), she worked on it with support from other preservice teachers, and the time to develop a specific plan (this was part of an assignment for another class where they developed a lesson plan).

In the moment of teaching I miss the implications of what Stella has proposed and return to Jeff's original challenge. I may have limited my thinking to the idea of social justice mathematics as social critique lessons. As we continue notice how, when I try to make this idea practical for Jeff, I recenter abstract mathematics. In response Jeff brings up the importance of "communication" (l. 147), which he views as facilitated by abstractness. While we have all critiqued the dominance of abstract mathematics, when it comes to what we might do instead, most of the time we end up recentering abstract mathematics.

135. Teacher: I think as well
136. so if you imagine to yourself
137. that one of these boatmen
138. is one of your students
139. lets say physics since you were a physics right.
140. So Stella talked about force how much force it takes to bend it
141. so if you try to teach this person about force from a traditional physics approach
142. how do you think that's going to go?
143. Jeff: I don't know.
144. Teacher: He's illiterate hasn't been to school.
145. Jeff: well obviously you aren't going to
146. but that's kind of what my whole caveat was here
147. is just communication
148. being able to communicate
149. so I would have a hard time teaching that person the way I learned physics
150. Teacher: But if you can learn

151. if you can talk you know
152. about his understanding about force
153. and then connect your traditional understanding of physics of force
154. to his understanding of force
155. he is going to grasp the traditional understanding better
156. than he would if you ignore the knowledge that he already has.
157. Jeff: I'm happy with that kind of conclusion
158. and then so the flag I would raise though
159. it which one of us is doing that connecting
160. well it's not the boatman.
161. Teacher: Right.
162. Stella: Right.
163. Jeff: That means something.
164. Teacher: That's part of the authority that you have as a teacher.
165. Jeff: It's not just authority
166. I'm saying that the approach the ability to abstract things
167. helps to communicate it
168. and that's what the boatman as good as he may be is lacking
169. it seems like
170. and so I'm not trying to downplay
171. that that's an important way of thinking about mathematics
172. it's just if we also want to communicate about mathematics
173. we need a way to do that.
174. Teacher: Sure. What I'm saying here as far as teaching though
175. that you welcome and you draw on those understandings
176. to make the teaching more understandable
177. ((overlapping talk in agreement))
178. Stella: I don't know never mind.
179. Teacher: And and this is where the critical aspect comes in
180. and that you recognize that the abstract traditional way
181. may not be any more valid than the way he understands it
182. but that it is needed for future classes for tests etcetera
183. cuz that's the standards that
184. Jeff: And from the standpoint of making boats totally
185. It's probably not just equally valid maybe better
186. because he can build boats right
187. so I mean I think it's important to keep straight
188. when you say valid there's a context that comes with that
189. so if we're trying to communicate about math
190. the word valid might take on a somewhat different
191. Teacher: Depending on who you are trying to communicate to.
192. [If you are trying to communicate to math teachers
193. Jeff: [Yes. I'm not trying to argue with you
194. but I am trying to be clear on what distinctions
195. because you know there is a reason why it isn't
196. maybe the one and only best way

197. but we do it the way we do it today
198. I mean the way I was taught math there was a good reason for it
199. and I don't want to discount that
200. while on in the quest for finding new ways
201. to also think about it, is that ok?

As I try to connect this example to a situation that is more real to Jeff (ll. 150-156), I do so in a way that subordinates the boatman's knowledge to that of traditional physics. I make the position of the dominant discourse clear when I close with how the boatman "is going to grasp the traditional understanding better" (l. 155). This suggests that this was the goal all along and that learning about using the boatman's understanding is just a trick to make the dominant discourse more palatable/understandable. Despite this (or because of it), Jeff brushes this idea aside ("I'm happy with that" l. 157) and focuses instead on who has to do that "connecting" (l. 159) work, suggesting that work is beyond the role of a mathematics teacher. Jeff then brings up the role of "the ability to abstract things" (l. 166) which is necessary "to communicate it" (l. 167). He sets up an implicit binary between "the boatman, as good as he may be" (l. 168) and a mathematics teacher/himself who has the "ability to abstract things." In this way the dominant discourses of mathematics divide people into groups. In this case the groups are mathematics people (who have the ability to abstract things) and nonmathematics people. These divisions then have implications for the perceived intelligence of members of each group. Like Stella, Jeff brings up the idea of communication (l. 172). However, in stating that "we need a way to do that [communicate]" (l. 173) he implies that abstract mathematics is the only way to communicate mathematically. Unlike Stella he does not separate the mathematical idea from the form in which it is communicated. As I respond (ll. 174-183) I continue to center abstract mathematics, while pointing out that alternatives to do this

are “valid” (l. 181). We are still unable to play with the dominance of a Western-centric perspective on mathematics.

Jeff picks up on my mention of validity (l. 185) and attempts to frame boatmen’s mathematics in more positive terms pointing to the relevance of context (l. 188). He suggests that within the boatman’s context his knowledge is more valid than ours (l. 185), while in the context of communication abstract mathematics is more valid (ll. 189-190). This idea is initially seductive pointing to the value that each form of knowledge has within its own context. The suggestion is that we can appreciate the value of each. However, like a colorblind perspective on race this perspective fails to acknowledge the power differences between the two contexts. That colorblindness (or something analogous) is in operation here is partially apparent by the lack of any mention (throughout this discussion) that the engineer (in Stella’s example) is likely White and upper-middle class, and the Indian boatman is Black and likely poor, even by his own country’s standards (also unacknowledged is the assumption that both are men). None of us point out the vast differences in power and prestige that are accorded to abstract mathematics over bodily knowledge or any other form of mathematical knowledge. We may have avoided this discussion of race by holding to the White ideal of race as irrelevant and to avoid breaking the progressive White taboo on bringing race into discussions. In all of our discussion both Jeff and I (and to a lesser extent Gavin and Stella) stay within the binary of pure (abstract, communication) and applied mathematics (boat making), in a way that reinforces the dominance of abstract mathematics, especially within a school setting.

Throughout this final section Jeff is clearly trying to reconcile the disagreement

between us (him and me). He shows this in his attempt to portray the boatman's knowledge in a positive light (ll. 184-190) and his disavowal of argument (l. 193). Even as he does so he reasserts the dominance and value of abstract mathematics. Jeff does this particularly when he states "there was a good reason for it [abstract mathematics]" (l. 198) and that he does not "want to discount that [abstract mathematics]" (l. 199). His use of the verb "want" (as opposed to "can't", "shouldn't", or "I don't know how", etc.) is important since this describes a desire on his part to keep the abstract mathematics in its dominant position. He closes with almost a plea ("is that ok" l. 201) that I not make him let go of abstract mathematics as we explore "new ways" (l. 200).

Teaching Social Justice in Mathematics Is Uncertain

We again focused on an entire class period on social justice mathematics in our week 12 class. This time we were challenged by the uncertainties of what it means to teach mathematics for social justice. During the intervening weeks (week 7- week 11) we spent time developing an understanding of action research, planning and implementing action research projects, and reporting on the progress of the action research projects. In our week 12 class the students read the work of Gutiérrez (2009), Trexler (2013), Gutstein (2012), and D'Ambrosio (1994) about teaching mathematics, social justice, and ethnomathematics. In this class students created a set of questions that they had related to the readings in particular and to teaching mathematics and social justice more generally.

The discussion below comes from a question that Jane asked about the social justice mathematics work proposed by Gutstein (2012) in which he integrates multiple projects into the mathematics curriculum. These projects dealt with elections, population displacement, and the spread of HIV/AIDS, among other topics. Jane was specifically

asking about the feasibility of doing this kind of teaching within a standard school mathematics curriculum. This question is not particularly surprising as it is a common response to social justice teaching. However, I was surprised that Jane was the teacher candidate who posed this question. Jane did her student-teaching in a school that had the most nonstandard curriculum of any of the teacher candidates. She and her mentor teacher regularly incorporated projects into the curriculum and often integrated their class with the science class. As Jane had recently begun applying for jobs, she may have been concerned about opportunities to do this kind of teaching in a more traditional school environment. I began the class by having all of the teacher candidates tell the rest of us what questions they had. I wrote these questions down and then led the class through a discussion of the questions. I begin this analysis by looking at Jane's question as she originally stated it and then look at our later discussion of this question.

1. Jane: And then my other one was
2. and this is from the last article
3. the math as weapon
4. he talks about how he uses current events to guide the math they cover
5. and I know this isn't necessarily a question about the article
6. and I don't know if you know this
7. but if he has a curriculum he has to follow
8. or if he just teaches what he wants to
9. Teacher: Yeah so
10. Jane: So can you teach that way with a curriculum to follow I guess?

The "he" Jane refers to in this section is Rico Gutstein. As Jane initially proposes her question she sets it up as a choice between following a curriculum (l. 7) and teaching "what he wants to" (l. 8). She uses "has to follow" (l. 7) suggesting that teachers have little choice if the school or district has adopted a particular curriculum. By posing this in opposition to teaching "what he wants to" she suggests that if a teacher has a particular curriculum then she will not be able to teach what she wants to. This idea of teacher

authority over what is taught is reflected in her reformulation of the question in line 10. Here the use of "can" places emphasis on the teachers' freedom to teach rather than on the teachers' ability to do so.

Later in the class, as we take up Jane's question, I reform it in order to begin the discussion. I explicitly mention social justice (l. 2), even though Jane did not, since social justice was the focus of Gutstein's (2012) article. I maintain the use of "can" (l. 2), and the discussion that follows begins with a focus on a teacher's autonomy to teach in this way. However, as Gavin and Esperanza attempt to understand why teachers feel restricted in their attempts to teach for social justice Gavin points to how, for many (including himself), social justice ideals are abstract and imaginary.

1. Teacher: so let's talk about this one
2. can you teach social justice in a standard curriculum
3. you were talking specifically about the Gutiérrez no the Gutstein article right
4. um so first of all does anybody want to answer that
5. based on what they saw in the article

Lines 6-58 ((we discuss restrictions based on curriculum, school environment, and administration expectations))

59. Esperanza: I'm going to say something
60. I talked to my administrator about this
61. and she seemed so for it
62. but I don't think that they're well aware of what it means
63. that's why they don't encourage it to all the teachers
64. Teacher: Ok.
65. Esperanza: My administrator was very like you should do that
66. that is awesome
67. but then she wasn't like supportive at all
68. Gavin: And I think like the biggest reason is
69. it does sound nice
70. but can they imagine it you know like
71. and that's one of the things where I have a hard time is
72. like I think these are great ideas
73. but as I try to imagine it
74. I have a hard time
75. because it's so there's so many uncertainties
76. that I can't really fully grasp
77. and I think that's the biggest thing

78. like stops these kinds of things from going on
79. is the people who are really in charge are like
80. oh that's a good idea but it's really improbable
81. so let's just throw it throw it out the window.
82. Jeff: Those are really abstract idea
83. the power identity what was the matrix you had
84. Teacher: Access achievement
85. Jeff: those are abstract I mean
86. what is power
87. when do you have it
88. when don't you
89. just because you've been given a title
90. doesn't mean they actually have the authority
91. and something else is going on
92. and when you talk about changing so that people have more power
93. that is just hugely situation specific
94. and you probably can't project that into the situation
95. it just has to happen somehow
96. I don't think you can plan for that
97. and it's very difficult to think about
98. Teacher: I agree I think it is very difficult

After I set up the discussion on Jane's question there is some brief discussion about general curriculum styles and school environments. When Jeff mentions administrators being opposed to social justice perspectives (in the section omitted from the transcript) it prompts Esperanza to begin her comment (l. 59). As she describes her experience with an administrator who seemed supportive she shifts the conversation away from a focus on restrictions and towards understanding those restrictions. From her perspective, these come from a lack of understanding on the part of administrators (l. 62) rather than an opposition to social justice.

Esperanza's characterization of her administrator's response sums up a general attitude towards social justice: we are "so for it" (l. 61) but not "well aware of what it means" (l. 62). Gavin agrees with Esperanza's comment and expands it to connect to the administrator's ability to "imagine it" (l. 70). He connects this same difficulty with social

justice mathematics to teachers (himself) since he sees social justice mathematics as "great ideas" (l. 72), but since he has to "imagine it" (l. 73) he has a "hard time" (l. 74). Gavin is pointing to the lack of examples of what it looks like to teach mathematics for social justice. As seen previously, playfulness in these moments may provide discursive space for teacher candidates to imagine new possibilities for mathematics education. However, we do not approach this particular moment playfully. Instead there is a sharp contrast to how the students have talked about teaching abstract mathematics (see previous discussion of teaching the quadratic formula), which is "real" to them. They can see it, see themselves doing it, and describe it in detail, because they have seen it done so many times. This is part of the dominance of these discourses. Thus teaching abstract mathematics is portrayed as certain, in contrast to social justice mathematics where "there's so many uncertainties" (l. 75). However, even though he had connected this challenge to teachers he then points to the "people who are really in charge" (l. 79) as responsible for stopping these ideas. In either case he sees the underlying problem as the uncertainty of social justice mathematics. Jeff builds on Gavin's point by naming social justice values as "abstract ideas" (l. 82), specifically citing "power identity" (l. 83) from Gutiérrez (2012c). For Jeff the application of these principles is "hugely situation specific" (l. 93) and this makes it "very difficult to think about" (l. 96). Jeff recognizes the complexity of teaching for social justice because of its dependence on context. However, for him this complexity becomes a reason not to engage in teaching mathematics for social justice. From this perspective (in response to Jane's question) a teacher cannot teach for social justice because "it just has to happen" (l. 95), "you can't project that" (l. 94), and you cannot "plan for that" (l. 96). This lack of ability (signaled

by the negatives in combination with can) is connected to the certainty provided by the discourses of abstract mathematics. Mathematics teachers can teach abstract mathematics because it is certain and they cannot teach social justice mathematics because it is uncertain. While this divide may make it seem as though the solution is to make social justice mathematics certain, but instead to imagine teaching that does not have to be (always) certain.

Gavin is expressing an unwillingness to leave the certain, comfortable world of abstract mathematics in order to enter the uncertain worlds of social justice mathematics unless/until he can "fully grasp" (l. 76) what it means to teach mathematics for social justice. For Jeff this uncertainty is expressed as an inability to grapple with the complexity of teaching mathematics for social justice. I can relate to what Jeff and Gavin express here and I do not believe that this fear or unwillingness is not unique to Gavin and Jeff, and it offers insight into the challenge of preparing mathematics teachers to teach for social justice. Answering this challenge is difficult, especially in the condensed time we have to work with teachers during teacher preparation. While we can model social justice teaching for our students and may be able to find examples of teachers who practice social justice teaching, these examples will still only be a drop in the bucket compared to the many examples of teaching abstract mathematics.

Further making the social justice mathematical world too certain for teacher candidates risks making a dominant form of social justice mathematics that is upheld as *the* way to teach mathematics for social justice. Thus as teacher educators we will need to balance developing a clear theoretical foundation and understanding of what teaching for mathematics can be with the flexibility to adjust to new contexts and new situations. Part

of this is incorporated in Gutiérrez's (2009) "equity stance." This equity stance includes the uncertainties of *not* knowing our students, *not* teaching mathematics, and *not* being in charge. The great challenge of this understanding is that the world of social justice mathematics cannot be made certain without destroying that world. The necessity of maintaining the uncertainty of teaching mathematics for social justice does not imply that teacher candidates should not observe teaching mathematics for social justice. Instead, as teacher educators we will need to prepare them to understand and respond to the complexity of teaching mathematics for social justice in a variety of contexts. This will likely include the development of a playful attitude towards teaching mathematics, especially a willingness to be a fool and an openness to surprise (Lugones, 1987). For White, male teacher candidates (like Jeff, Gavin, and myself) this may be particularly difficult. Playfulness can prepare teacher candidates to travel to the uncertain worlds of social justice mathematics without trying to make it certain or imposing their own rules on that world.

The uncertainty of how to teach mathematics for social justice was illustrated as we later turned to one of Gavin's questions. Specifically Gavin asked about how to involve students in the creation of and decisions about social justice mathematics lessons. In response I direct them to consider a reading they had done from Gutstein (2012). During this discussion note how our knowledge of mathematics as abstract impedes our ability to envision how to connect social justice topics to a standard mathematics curriculum.

1. Teacher: So how did Gutstein talk about that
2. do you remember?
Lines 3-16 ((some talk as we get to the right page in the article))
17. Lisa: Oh surveyed and asked topics they were interested in

18. Teacher: Yeah so the topics students came up with were
19. criminalization of youth of color
20. and sexism or sexism was one that he proposed
21. um and then going farther down
22. through dialogue we collectively agreed on 5 topics
23. elections, displacement including gentrification, foreclosures and immigration
24. deportation HIV/AIDS criminalization and sexism
25. Jeff: And so they were going to investigate these with mathematics
26. Teacher: Yeah
27. Jeff: But how would so even as I read the list
28. I'm thinking I don't even know what the appropriate mathematics
29. would necessarily be
30. so how can you do that first
31. Teacher: Right
32. and that's where
33. so if we take something like elections
34. I think we have some idea of the mathematics that would be involved
35. there you've got polls so statistics
36. Jeff: Probability and statistics
37. Teacher: Yeah displacement gentrification foreclosures
38. we probably have some ideas there
39. loans interest rates and some of those kinds of things
40. immigration and deportation we may not have as much of an idea
41. um as far as what mathematics would connect to that
42. but you can look at demographic data so it's more of a statistical kind of thing
43. Jeff: Its descriptive
44. Teacher: The HIV/AIDS that's probably one of the harder ones
45. at least for me to think about mathematically.
46. Jeff: But these all kind of fall into a statistical realm
47. I mean right don't you think
48. Gavin: That kind of is where my question is stemming from is
49. like I understand that you can get these kinds of topics to talk about
50. and have these things done within math
51. but I feel like at the moment with
52. where I'm at in precalculus
53. I don't I'm having a harder time like
54. Teacher: Well so my understanding of the common core
55. you're doing some statistics in every year right
56. Gavin: yeah but my class is
57. Jeff: Up through math 3
58. and then precalculus goes back to the standard
59. Gavin: different when I'm actually teaching a secondary class
60. compared to a precalculus class
61. and so like right now within the context I'm in
62. I'm struggling with this aspect
63. Teacher: didn't he talk about

64. with the HIV/AIDS that the actual models are done
65. with differential equations
66. Jeff: Well they would be for sure um
67. Teacher: Ok so that's beyond precalculus right um

As we begin this discussion Lisa points out that students were surveyed about topics (l. 17) that they were interested in, and I list those topics as elections, displacement, HIV/AIDS, criminalization, and sexism (ll. 23-24). Following this Jeff jumps right to the crux of this issue which is that we "don't even know what the appropriate mathematics would necessarily be" (ll. 28-29). The uncertainty that Jeff highlights here is the ability to determine which mathematical ideas will be useful in addressing the topics selected. This is implicitly contrasted to a standard curriculum where every topic to be covered in a particular unit is specified and, in some cases, teachers are told what mathematical topics they should address day by day. To be clear, each of these students was selected for these positions based on her or his mathematical ability and will shortly have an advanced degree in mathematics. On the whole their mathematical knowledge is advanced. Yet because of how mathematics is conceived of and taught in schools, if Jeff is correct, then they do not know how to connect their mathematical knowledge to these real-world topics. This again is part of the pure/applied binary. Jeff and Gavin, are trying to think of a way to apply the pure mathematics that they were taught to a specific social justice situation. I experienced similar struggles in my own teaching. However, learning to apply mathematics has not been part of their (or my) mathematical training or part of their teacher preparation. In this way the dominance of abstract mathematics impedes their ability to teach for social justice.

I, in part, viewed my role in this class as making the social justice aspects of mathematics seem concrete and doable to them, so I step in at this point to try to make

some of these connections. I go first to statistics (l. 35) and while I mention "interest rates" (l. 39), statistics is what gets emphasized (l. 35, l. 42) and Jeff notes that "these all kind of fall into a statistical realm" (l. 46). Gavin agrees with Jeff, but also extends his comment to point to the difficulty he has in applying them to his particular class, "precalculus" (l. 52) This points to another problem within mathematics teacher preparation—that of the separation between the various branches of mathematics. This separation is an extension of the dominance of pure mathematics. Each subject area has become so specialized that a mathematician will typically focus entirely on one small branch of mathematics. A more generalized, mathematically integrated approach is difficult to imagine from this perspective. Thus for these teacher candidates algebra, statistics, and precalculus are viewed as separate and unconnected topics. These separations are maintained by the level of abstraction within each and by how they have been taught. These separations appear to make it more difficult for preservice teachers to overcome the uncertainties involved in teaching mathematics for social justice.

Teacher Control and Student Resistance

In preparation for our class during week 13, I designed an activity to confront the idea of abstract mathematics as necessary for communication. After the teacher candidates and I discussed the mathematics of maps, I introduced them to the idea of Marshall Island stick charts. These charts allowed the Marshallese to navigate between distant islands by following patterns in the ocean swells and without modern navigation equipment. However, each chart was individualized such that only the maker and a select few others would be able to read it. I made the argument that this is similar to modern mathematics in the sense that the notation and symbols are only accessible to a relative

few people worldwide. As the discussion shifts to compare the Marshallese stick chart makers to modern mathematicians, the teacher candidates point to the elitism of mathematics rather than its universality, momentarily. Then discussion quickly turns back, suggesting that the exclusions caused by abstract mathematics are natural.

I struggled during this lesson to let go of some of the control over the direction of the discussion. As I posed questions to the students I was clearly looking for certain "correct" answers. I was not playful in my approach to the students and they likely did not feel that they could be playful in response. As a result the transcripts below (including the portions I've omitted) are dominated (in terms of number and lengths of speaking turns) by the males (including myself). In addition the number and length of my speaking turns are much greater than in week 4 which was analyzed earlier. All together these characteristics seem to have led to a discussion where the students (Jeff in particular) are posing greater resistance than during other discussions. The repeating pattern in these discourses are that I will attempt to prove a point by logic, Jeff (or others) will appear to agree with the logic, but then quickly turn back to their original point of disagreement—that the divisions created by mathematics are natural.

1. Teacher: Ok um ok so pre world war two
2. people who live in the Marshall Islands made these stick maps or charts.
Lines 3-53 ((we discuss how the stick charts work))
54. Teacher: Um how is that kind of keeping that information
55. specific and individual
56. how is that like mathematics?
57. We've talked a lot about mathematics being universal
58. but I want to talk about it the other way
59. how is it not universal
60. how is it more focused I guess more individual or restricted
61. to a small group not individual
62. Lisa: There is like a sense that
63. there's some mathematical experts
64. and they will share what they feel is important with others

65. Jeff: So you're saying like the map guide
66. that they can read waves same kind of idea
67. Lisa: Yeah like they choose to share it
68. Teacher: or not
69. Lisa: And they choose what's important to share
70. Teacher: Go ahead Gavin.
71. Gavin: We kind of talked about it
72. over the conference up at Logan but it's kind of like a gate keeper
73. I don't know maybe it was a different one
74. but one of them it talked about how like algebra is like a gatekeeper
75. like these students could learn algebra disregarding everything
76. they could potentially go and make them eligible for college or not
77. Jeff: So I mean I can see this a lot in secondary
78. Teacher: What's that?
79. Jeff: Well you know
80. like we'll say ok this student will probably do x
81. so we push them in that direction
82. but I mean at some point
83. when you get to a certain level of expertise
84. I mean it just takes a really long time to get there
85. it's not like they're gatekeeping it's just a fact of life
86. they it takes you that long to learn it and that's all there is to it
87. I mean it's a different reason but it has the same effect
88. Teacher: Well I think the gatekeeping part of it is
89. um does not passing algebra II really mean a student can't be successful in college
90. Jeff: It depends on the field obviously
91. but it doesn't mean that they can't be
92. Teacher: Right and I think that's where more of the gatekeeping comes in
93. when there's not necessarily a direct correlation
94. between that specific knowledge and whatever is required for success
95. at whatever the next level is
96. and algebra II is kind of the key one there
97. if you're through algebra II
98. usually if you go beyond algebra II you're going to college
99. if you don't you're probably not.

Lines 100-149 ((we agree on the relatively small number of people who can communicate with abstract mathematics through a comparison to language))

Lines 150-216 ((Jeff, Gavin, and I continue going over some of the earlier points about gatekeeping))

When I pose the original question ("how is that like mathematics?" l. 56) I do so in a way that is open to multiple responses and that would allow the teacher candidates to respond in a variety of ways. However, as I continue I reframe the question, essentially telling the

teacher candidates what kind of answers I am looking for ("I want" l. 58). The final version of the question is best summarized as "how is it [mathematics] not universal?" (l. 59). I have answered my own original question by telling the students that the stick charts are like mathematics in their nonuniversality, thus attempting to limit their answers to those that agree with the nonuniversality of mathematics. I further specify that I am looking for answers that show how mathematics is "focused" (l. 60) and "restricted to a small group" (l. 60-61). The term "restricted" in particular suggests that I am looking for the negative effects of mathematics. Lisa dutifully responds to my question. She points out that there are mathematical experts (l. 63) and she attributes authority to them to distribute information (l. 64) or not. However her use of the word "sense" (l. 62) shows hesitancy in her answer, suggesting that she may not be completely comfortable with this response. After Jeff and Lisa clarify what she means, Gavin connects this idea to the gatekeeping (l. 72) that happens in mathematics courses, especially algebra (l. 74). Gavin's implicit critique is that "these students could learn algebra" (l. 75), but they do not and they could "be eligible for college" (l. 76), but are not. Notice that these statements lack an agent and no one is made responsible for these decisions that keep students out of algebra and out of college. The lack of agent helps to avoid a conversation about what these teacher candidates might do to overcome the effects of tracking. This answer also fits what I was suggesting to them as I reframed my questions.

To this point I have largely stayed out of the discussion after posing the question. Allowing the students to run the discussion has not been a problem, since both Lisa and Gavin gave answers that matched what I was looking for. However, as Jeff promotes a position that I did not agree with, I take a more active role to push the discussion back

where I want it to go. Jeff acknowledges the gatekeeping (ll. 80-81) that Gavin mentions, but then he defends it, beginning with "but" (l. 82). From Jeff's perspective mathematics requires "a certain level of expertise" (l. 83) and because it "takes a really long time" (l. 84) it is not really gatekeeping "it's just a fact of life" (l. 85; see also "that's all there is to it" l. 86). Jeff is presenting the way that mathematics courses divide students into college and noncollege tracks as natural and inevitable. This removes responsibility from the teachers and other decision makers and makes it a *natural* part of mathematics. However, this perspective is only possible when mathematics is understood as *naturally* abstract (and so separated from the lives of the people who use it), and as a consequence neutral and impartial in these political decisions of who goes to college and who does not. With "well" (l. 88) I express my disapproval of Jeff's answer and then ask him a question that has only one "correct" answer (l. 89). Jeff hesitantly gives the "correct" answer I was looking for (ll. 90-91) and I quickly step back in to evaluate his response ("right" l. 92) and retake control of the discussion, giving a more complete version of the answer that I was looking for from the beginning. As this discussion continues the same pattern repeats multiple times. I ask closed questions, someone will give the "correct" answer, Jeff will reassert these divisions as natural, and I try to push him to my point of view. Throughout these sections Jeff, Gavin, and I dominate (all White males taking over) the discussion until we run out of time for this discussion and the teacher candidates each report on the progress of their action research projects.

Just prior to the beginning of this discussion I made a brief comment to myself about not having enough time. As a result I seemed to have tried to rush the teacher candidates to see and agree with what I wanted them to get out of the activity we had

done. This led to a situation where I attempted to use the authority of my position (teacher, White male) to convince the students of my position. I was not playful and the teacher candidates could not be playful without risking upsetting the power dynamics of the class. There had been moments prior to this lesson where the teacher candidates had pointed out my overt use of authority to guide the discussion, but that did not happen in this class. Instead some of the teacher candidates went along with what I was trying to do (supplying the correct answers), possibly because they already agreed with me, while Jeff openly resisted. However, it is unclear if he was resisting my use of authority, the ideas themselves, or some combination of both. Under different circumstances this activity could be used playfully with different results. However, since I asked questions that I already knew all the answers to, setting up my judgment of their responses, there was no opportunity for surprise either on my part or for the teacher candidates. Additionally responding playfully would likely be considered unsafe in this situation.

Conclusion

As we discussed teaching mathematics and social justice the issue of the abstract in school mathematics quickly became apparent. This is not surprising given the way that abstractness is privileged in school mathematics (Walkerdine, 1990) and how political and contextualized social justice mathematics needs to be (Gutiérrez, 2002a). This school mathematics world is dominated by the discourses of abstract mathematics. These discourses that operate from the basis of a pure/applied mathematical binary limited our ability to imagine a socially just mathematics. Additionally, what this analysis reveals is that part of the challenge in working with secondary mathematics teacher candidates is that they often feel a need to defend and maintain abstract mathematics, because that is

part of what it means to be a “good” mathematics teacher. In many cases they feel this pressure from their mathematics professors, from their mentor teachers, from their students, and from their peers (both mathematical and not). Their struggles (which will be the theme of Chapter 7) come in part because of the multiple and contradictory expectations they face.

Not recognizing these expectations and pushing teacher candidates to agree with our point of view can create moments of greater resistance. This may be especially important for White male teacher educators who may be less aware of their use of authority to push ideas. Unless we (as teacher educators) can create spaces where the teacher candidates can engage with these ideas playfully (and not defensively) then we will make little progress in teaching mathematics teacher candidates to think about mathematics education differently. Many of the transcripts analyzed in this chapter hint at the importance of how these teachers position themselves in relation to mathematics, social justice, and their students as well as their authority or lack of authority in these positions. This will be the theme of the following chapter, but these positions are often constructed in relation to abstract mathematics.

CHAPTER 6

SUBJECT POSITIONS: TEACHER CANDIDATES AS POWERFUL AND HELPLESS

Mathematics teachers have invested years and money in developing mathematical competency and learning how to position themselves as mathematically competent. As master's students, the teacher candidates I worked with have invested additional time and money developing their mathematical competency. As a result of these investments these teacher candidates may feel a need to protect and preserve the value of their investments by asserting their mathematical and pedagogical competence. As explained in Chapter 5 investments in abstract mathematics complicated our efforts to understand teaching mathematics for social justice. Teaching for social justice may be perceived as a threat to our mathematical competence, because of its contextualization and because it does not fit what it traditionally means to be a "good" mathematics teacher. This chapter explores how these teacher candidates positioned themselves and their students in relation to each other and in relation to mathematics. Frequently these positionings preserved our investments in dominant discourses, and at other times, by positioning students in more complete ways we could work against the dominant discourses of mathematics education.

Martin (2011) characterizes mathematics education as a White institutional space based on the general exclusion of people of color from positions of power, policies that favor Whiteness, a curricular model developed by White elites, and a view of the subject

matter (mathematics) as neutral. The mathematics teacher candidates in my class were well versed in the logic and discourses of the White institutional spaces of teacher education and of mathematics education. As a result I expected to see the influence of Whiteness from all of them regardless of their racial or linguistic background. To some extent learning and using these discourses has been necessary for them in order to achieve as they have in college and in mathematics in particular. Despite my efforts to make my class a safe place for race talk and social justice (the problem of safety will be discussed later) talk these classes still took place within a White dominated class (five of the seven teacher candidates, along with the instructor are White), within a White dominated institution, and was sanctioned both by the mathematics department and teacher education program. Thus normative ideas of what it means to be a "good" mathematics teachers were likely to be the standard of interaction and expectation on multiple levels.

According to Yoon (2012) in order to preserve these investments in Whiteness, White teachers will often project an image of helplessness in the face of critical or uncomfortable conversations. This "helplessness" serves to release the individual from taking blame while allowing him or her to acknowledge injustice. In this way the teacher can maintain a self-perception as "good," by deflecting potential accusations or feelings of guilt. However, because the teacher candidates I worked with were student-teachers (and I as their supervisor) they needed to also project an image of competence. Helplessness and competence can be thought of in terms of varying degrees (or different kinds) of authority. Thus the overarching work that teacher candidates are doing in the following transcripts is to position themselves and their students in terms of their

authority. They may portray themselves as authoritative in terms of mathematical knowledge and ability and/or in terms of being in control of the classroom.

This authoritative positioning for teachers is often made by positioning students as helpless in order to justify their use of authority, reflecting a discourse of authority as necessarily authority *over* students. Within this discourse a mathematics teacher's authority is closely connected to her knowledge and ability with abstract mathematics. This authority is connected in at least two ways to abstract mathematics. This connection occurs because abstract mathematics is the key marker of a mathematics teacher and because of the perception of neutrality that accompanies abstract mathematics. In contrast, teachers may portray themselves as helpless (lacking authority) in terms of correcting or addressing social injustice. These various positionings seem necessary in part because of the precariousness of being a student-teacher and the need to demonstrate competence (goodness) in terms of mathematical ability, teaching ability, and to maintain control (authority) over a class of high school students.

Discourses of Responsibility and Authority

Dominant discourses in education frame teachers as responsible to the school, to their discipline, to their peers, and possibly to society. Typically this responsibility is connected to some kind of perceived debt (i.e., the teacher "owes" something to their school for employing them). From this perspective responsibility is used to justify requiring teachers to engage in prescribed practices. These discourses of responsibility are based in part on discourses of individuality and meritocracy. These same discourses are used to justify the vast inequities in schools by suggesting that students would be able to overcome these inequities with more motivation or greater effort (Gutiérrez, 2015).

Together these discourses focus attention on where to assign blame, rather than on addressing inequities and creating new possibilities for education. Political leaders blame school district leaders for poor educational outcomes, school district leaders blame principals for not getting teacher buy-in for the newest change to the education system, and teachers blame students for not doing homework or not trying. While students may provide more interesting arguments about why inequities exist and what to do about them, their critical perspective has no place in these discourses. Within this hierarchy of blame, teachers, to the extent that they are seen as responsible to their school, district, etc., are seen as responsible *for* their students. Responsibility for students then becomes a need for teachers to exercise control over what students learn and how they behave in their classes. These discourses position teachers as agents, often the only agents, in the classroom.

Abstract mathematics plays a key role in maintaining the dominance of discourses teacher responsibility and is a means of maintaining control in the mathematics classroom. As explored in the previous chapter the perceived neutrality of abstract mathematics makes it easier for the mathematics teacher to pass blame onto the students, than teachers in other subject areas. These discourses though also depend on particular positions for teachers and students. These discourses require that teachers exercise authority in order to gain compliance from their students. While the methods of exercising authority are varied, they should, according to the discourse, lead to students who follow instructions, including following prescribed means of classroom participation. Although teachers may accomplish this level of compliance through a variety of mechanisms, including some that may be considered fun or exciting (in relative

terms), the overarching theme is one of control: The teacher must always be in control of the classroom. Abstract mathematics provides a means of control to mathematics teachers. Mathematics teachers can control acceptable content and methods of participation by judging the "correctness" of student work or through demonstrations of their mathematical ability. These demonstrations can be used to prove to students the apparent superiority of the mathematics teacher's knowledge, and thus provide justification for why the students should comply with the teacher's requests.

In our discussions of mathematics teaching we were regularly caught up in these discourses of responsibility and authority. Margonis (forthcoming) suggests that teachers can opt out of these discourses through responsiveness. Through responsiveness, instead of blaming students (as dominant discourses would have us do), we are open to the messages students send, whether through words or actions, and that we have an ethical obligation to be responsive to those messages. Responsiveness requires that we view our students (and by extension their messages) as worth hearing. Along these lines, there were times when, in our discussions, we proposed a different kind of responsibility. I call this conception of responsibility, responsibility *to* students.

My understanding of responsibility to is similar to Heldke's (1987) notion of the coresponsible option. While Heldke is suggesting a relationship between inquirer and object of inquiry, I believe her description fits for teachers and students as well. She describes the coresponsible option as a relationship between two people who have "*responsibilities* to each other, obligations to treat each other with respect and care" (emphasis original, p. 129). This suggests a shared responsibility and thus a shared authority that counters the dominant discourses suggesting that in a teacher-student

relationship only one party can hold authority. Further a coresponsible relationship requires that all parties inhabit subject positions as intelligent and capable of responsibility. While our class discussions largely focus on the teacher part of this relationship, it is only when we position students as intelligent and capable partners in education that we can also position teachers as responsible to them. Unlike dominant understandings of teacher responsibility, responsibility to students is not based on the idea of a debt owed. Instead this represents our attempts to create an alternate teacher subject position in relation to authority by repositioning students as capable learners and worthy of respect. In repositioning students in this way teachers are necessarily repositioned to more regularly work with (alongside of) students rather than to only exercise authority over.

This responsibility to students channels the use of teacher authority in ways that promote the students' best interests and suggests that teachers are accountable to their students. The idea of responsibility to students incorporates Gutiérrez's (2009) equity stance. This equity stance includes maintaining a tension (not choosing one over the other) between being in charge and not being in charge in the classroom. From this perspective a teacher attempts to balance students' needs and desires in considering what is taught and how it is taught, and works to involve all students. However, there are also limits to this authority which may mean that some students choose not to engage, and that the teacher cannot force engagement (which dominant discourses push). This approach to mathematics teaching requires that students be positioned as participants in their own education.

In this chapter I continue to draw on selections from our transcribed class

meetings. I focus on those moments when we discussed the roles of teachers, the roles of their students, or the relationship between teachers and students. In analyzing these moments of discussion I draw on Gee's (2005) building tasks. In particular I use the building tasks of "Identities" (how the speaker positions him- or herself and others) and "Relationships" (how the speaker constructs relationships between her- or himself and others and between others). Previous chapters used others of the building tasks, but these two are the most relevant to show how the teacher candidates and I positioned ourselves (and students) in relation to mathematics and social justice. Where possible I also connect our discussions to the beliefs and values that we draw on. This chapter is divided into two sections. The first section analyzes how we took up and were influenced by the dominant discourses that frame our ideas about responsibility and authority generally. The second section analyzes our attempts to define and understand responsibility to students.

Dominant Discourses of Authority and Responsibility

More Knowledge = More Power

Our class in week 4 focused on the discourses that shape our world as mathematics teachers and that shape the worlds of our students in our classes. The discussion below comes early in this fourth class and for these teacher candidates is their first introduction to the ideas of discourse and dominant discourse. During this discussion we talked about how teachers and students both have to conform to particular roles in order to be recognized as teachers and students. In this way power is exercised through discourses that structures teacher-student relationship in ways that conform to standard ways of taking on the role of mathematics teacher and mathematics student. In response to this previous point, Jeff begins this discussion about the power and authority that a

teacher has. In the process he positions himself/teachers in terms of power differences as compared with students.

1. Jeff: One thing just brought up a question I while I was reading.
2. You it's inevitable I mean if I I am more knowledgeable than my students
3. you can't change that
4. that automatically gives me some more power that they don't have,
5. you can't change it
6. so that's not necessarily a bad thing.
7. Sometimes there's this connotation that we want to remove all power differences.
8. Teacher: Right
9. Jeff: Um I think that that's erroneous.
10. Teacher: And I think to pretend that you don't have that authority is false
11. and the students are going to recognize it right away
12. and they're not going to respond in a way to it.
13. Jeff: Well and if they don't respect the fact,
14. if they don't think you have the knowledge
15. you can't teach them in any way
16. Teacher: Right.
17. You were going to say something.
18. Lisa: In this example both students student groups and teacher group
19. has like agreed to this to be in this relationship together.
20. Teacher: Yes. Yeah.
21. Gavin: So is that a discourse within the two discourse
22. like an overlapping of the two discourses [
23. Teacher: [Yeah, this this discourses overlap a lot.
24. Yeah.
25. The teacher discourse doesn't exist without the student discourse.
26. Right they work together.
27. Jeff: I'm not sure I there's tacit agreement.
28. Teacher: Yeah.
29. Jeff: They haven't actually thought about it probably.
30. Teacher: No.
31. Jeff: They don't think they have a choice,
32. so I don't think that they've agreed
33. they just ceded you know.
34. Teacher: Yeah.
35. They're still there.
36. Jeff: They probably don't know that it could be any other way you know

Lines 37-54 ((Jeff and I discuss what "dominant discourse" means.))

55. Teacher: We'll just call it dominant discourse. Is that ok?
56. Jeff: But I mean
57. I don't think I mean
58. I understand that how a student though
59. could come in and not have

60. I mean they're inexperienced
61. and it's this way for almost everything in their whole life
62. they don't know any different about most things
63. so it seems much more relevant for a teacher
64. who has been teaching for say 10 years
65. and who would like to change something
66. and who know what things are like now
67. and who has an idea of what might be better
68. and then can't or then runs into walls or whatever that's different

Jeff's opening statement carries a number of assumptions that position both students and teachers in important ways. He is drawing on a discourse that links authority to knowledge. Within this discourse greater knowledge necessarily creates greater authority. In drawing on this discourse Jeff positions teachers as naturally having more authority than students since he (and teachers generally) have "more knowledge" (l. 2) than students. Presumably he is talking specifically about mathematical knowledge, but also about knowledge more generally (this is more clear when he references students' "inexperience" later, l. 60). This knowledge differential is unchangeable ("inevitable" l. 2 and "you can't change it" l. 3, l. 5). Together these statements naturalize the relative positions of teacher and student and the teacher's authority as authority over (indicated by "ceded," l. 33) students, which is "automatic" (l. 4). Finally he evaluates these positions as "not necessarily a bad thing" (l. 6). Among the assumptions that are made within this discourse is that authority is static (i.e., is part of being a teacher), teacher authority and student authority are in an inverse relationship—as a teacher's authority increases, students' decreases, and vice versa, and that the authority-knowledge link is applicable only to school sanctioned knowledge—especially mathematics.

These assumptions set up the rest of our discussion and how we position teachers and students. This knowledge-authority link and the assumed necessity of exercising

authority over students requires that mathematics teachers regularly reassert their mathematical knowledge in order to be "good" mathematics teachers. In this case, being a "good" mathematics teacher means maintaining control over the class, because it is this control that allows the teacher to teach. This perspective is built on a view of knowledge as static. Mathematical knowledge is something that the teacher possesses and can then transmit to students. From this perspective, student participation and contributions are largely unwelcome, unless they match what the teacher expects them to contribute. Thus it becomes important for the teacher to use authority to keep students quiet and to restrict their contributions. The teacher also has to demonstrate his greater knowledge both through demonstration and by judging which student contributions are valid. This is often accomplished through the use of abstract mathematics.

While Jeff characterizes the desire to remove power differences as "erroneous" (l. 9), I (while apparently agreeing) change this to emphasize that teachers "pretend" (l. 10) to not have authority and that when they do it is "false" (l. 10). In this way I try to get the teacher candidates to question the way that authority is typically used. Jeff picks up on my suggestion of the agency of students ("they don't think you have the knowledge" l. 14) and then ties "knowledge" (l. 14) to authority and a teacher's ability to teach (l. 15). Here Jeff is using the idea of discourse that we have discussed to suggest that unless a teacher demonstrates superior knowledge students will not recognize the teacher as capable. From this perspective it becomes necessary for teachers to reassert their superior knowledge or the students will not let them teach. Students' potential to use their agency is used to justify an authoritarian approach to teaching. This is based on the assumption that only one (teacher or students) can have authority. Lisa suggests agency for both

parties in this classroom relationship and that they have in a sense "agreed" (l. 19) to this relationship. Jeff qualifies this characterization by noting that the agreement is "tacit" (l. 27), that they (students) "haven't actually thought about it" (l. 29) and "they don't think they have a choice" (l. 31). In this way he positions students as having limited capacity to think and understand their world. This positioning helps to justify a concentration of authority in the teacher. Students are positioned as having insufficient knowledge, and so cannot have authority, while mathematics teachers are positioned as more knowledgeable, which naturally leads to their greater authority.

To push the thinking of the teacher candidates I link these statements to the idea of dominant discourse (in the skipped portion of this transcript). In his response Jeff positions students primarily as "inexperienced" (l. 60). As a result students are unable to challenge dominant discourses because they do not have the experience necessary to think of alternatives. In this view student capability is limited by their perceived lack of knowledge. Jeff suggests (ll. 60-62) that knowledge about discourses is not relevant for students. Then he shifts to suggest (ll. 63-68) that experienced teachers ("teaching for say 10 years" l. 64) could benefit from a knowledge of discourses. By contrast he is implicitly suggesting that teacher candidates do not have the experience (l. 64) or the knowledge (l. 66) to effectively change a discourse (l. 65). This is a continuation of the discourses linking knowledge to authority. However, this time this link is used to justify new teachers in not trying to make changes to mathematics education. At the beginning of this discussion teachers were positioned as naturally authoritative, because of their greater knowledge, however, by the end (new) teachers are mostly helpless, because of their lack of experience. In this way connecting authority to knowledge allows teacher

candidates to position themselves as naturally more knowledgeable than their students (justifying teacher authority over students) and as lacking the knowledge (authority) to make significant changes in the classroom.

Who Benefits From Authority?

Later in this class we discussed who benefits from the exercise of authority through abstract mathematics. During this class Karl asked his classmates about the implications of performing a complex, abstract derivation of the quadratic formula; this discussion was analyzed in Chapter 5. The transcript that follows below is the continuation of that discussion. Here I shift the discussion to ask that if we consider the possibility that students do not benefit (arguably our conclusion to his question) then who does? Who benefits from that kind of teaching (formal, lecturestyle, mathematically abstract)? As we engage in this conversation about who benefits from abstract mathematics, the dominant discourse of a mathematically determined hierarchy of intelligence shapes the way that we position students.

1. Teacher: K. I'd like to go to this question now,
2. so thinking about deriving the quadratic formula the way Karl described it.
3. Esperanza: Completing the square?
4. Teacher: Um lecture style
5. doing it up on the board that kind of thing.
6. Who benefits from doing that?
7. Who benefits from deriving the quadratic formula?
8. So Karl is saying basically students didn't benefit from it.
9. Karl: When I did it
10. no
11. less so than other things I've done.
12. Teacher: K.
13. So who benefits from it?
14. Because this is a fairly common thing I think in math classes to do that
15. Jeff: I don't know
16. I think the teacher gains
17. you know feels kind of cool to do it

18. but other than that.
19. Lisa: Yeah.
20. Jeff: I don't think it has a lot of benefit.
21. Esperanza: I. Go ahead Stella.
22. Stella: No.
23. Esperanza: Even though we're saying like
24. none of your students actually benefit
25. I would argue that
26. because I have like a brilliant student in my secondary II
27. that even though [it isn't one of those
28. Teacher: [You can make an argument
29. Esperanza: lower level classes.
30. Teacher: that a few students benefit from it but
31. Esperanza: Ok yes you can't get that benefit from that
32. when you just say use the formula
33. they will not get that exposure so you have to keep their needs as well
34. and that's one way of doing it.

The first several lines here (ll. 1-14) are establishing the basic premises of the question as Karl had originally presented the idea. This frames the discussion around the assumption that students mostly do not benefit (this assumption will be questioned later) and focuses on who does benefit. In response Jeff points to the "teacher" (l. 16) who "feels kind of cool" (l. 17). This positions the teacher as someone who enjoys abstract proofs and the feelings of mastery that come with them and is connected to the idea that for "good" mathematics teachers doing mathematics is fun.

After a few brief comments Esperanza questions the idea that students do not benefit and brings up a "brilliant student" (l. 26) in one of her classes as someone who could benefit from it and that such a student deserves that opportunity. Her statement equates brilliance with an ability to understand and appreciate abstract mathematics. In this way she positions this particular student in a positive way, and implicitly positions other students in a negative way, based on their ability to appreciate abstract mathematics. Singling out this one "brilliant" student implies a continuum of other

students who are less than brilliant. This appears to be connected to a larger societal belief about mathematical ability as an inherent ability that students either have or do not, and which equates intelligence with mathematical ability. This discourse reinforces the idea of a mathematical hierarchy, which helps to maintain social inequality by hiding the normalized way that abstract mathematics divides students (Skovsmose & Valero, 2001).

As we continue Stella places students within a mathematical hierarchy, but also questions the validity of whether that is sufficient justification for a potentially damaging teaching practice.

35. Teacher: Stella.
36. Stella: Um yeah I was going to say also
37. a few students may ((benefit)) who like easily memorize and like to deal with algebra
38. instead of connecting it to geometry or whatever there is a connection you know
39. what I mean um but I wonder if that's a good decision if only a few students benefit
40. and it's a detriment to all the other students
41. like why would you even do that and that's kind of like
Lines 42-49 ((Gavin and I have a brief side conversation)).

Stella also references a mathematical hierarchy by pointing out those students who "easily memorize" (l. 37). However, in doing so she links it to student preference ("like to deal with algebra" l. 37), rather than to innate intelligence. She then questions the validity of this hierarchy in determining where mathematics teachers spend their time and energy (ll. 39-41). She positions teachers as people who need to make "good decision[s]" (l. 39) and who must consider the needs of the whole class (ll. 39-40). In this way she suggests other considerations that lie outside the discourses that create this mathematical hierarchy, but does not challenge the existence of the hierarchy itself.

As we continue the discussion I ask the teacher candidates to give some more specific answers as to how performing a formal, abstract proof benefits the teacher. Notice how the teacher candidates and I position teachers and students and frame the

discussion in order to make it safe for White teachers to engage in.

Lines 50-82 ((I explain why I think this discussion is worth having))
83. Teacher: possibly the main person that benefits from this is the teacher.
84. What benefit does it give the teacher?
85. Jeff: Practice
86. Teacher: Ok. So mathematical practice.
87. Jane: Authority.
88. Teacher: Authority.
89. How does it give the teacher authority?
90. Jane: It shows that you know what you're talking about.
91. Teacher: K.
92. You demonstrate how much greater your mathematical ability is than the students. K.
93. Lisa: You hold them down and confused.
94. Multiple: ((laughing))
95. Teacher: So there's there's something
96. so I I would argue that is from my perspective
97. the main reason that that continues to be done in math classes
98. um that by using abstract formal symbolic language
99. that is beyond the reach of most of the students that you're doing it to
100. that you're showing it to
101. it does confuse them
102. it kind of raises up your authority your knowledge in those kind of things
103. and so it's a way of reinforcing your authority as a teacher potentially.
104. Now does that mean as a teacher
105. Karl in this case
106. that he was thinking of those kinds of things?
107. Probably not but I think that is one reason
108. where you get resistance to changing some of these things
109. is because it can be a threat to that authority um
110. that can be a comfortable place for those of us who are comfortable with math
111. comfortable with those symbols and manipulating them
112. it's nice to have that authority
113. but it there may be some detriment to our students
114. and that's something to consider.

In response to my question of how the teacher benefits from performing an abstract proof in class, Jane quickly links it to a teacher's authority (l. 87), because it shows "that you know what you're talking about" (l. 90). As Jeff did previously Jane's comment connects a teacher's authority to his or her mathematical knowledge, which is assumed to be the knowledge of school mathematics. In order to be recognized as "good" (authoritative)

mathematics teachers, teacher candidates feel that they have to demonstrate their mathematical prowess, which helps maintain a racial hierarchy of achievement commonly found in mathematics classes.

In my response I expand Jane's comment and give it a meaning that she likely did not intend. While her statement assumes that teachers need this knowledge, my statement (l. 92) tries to show how this knowledge may be used negatively, in a way that is detrimental to students. In this way I position teachers as potentially using authority in ways that are damaging. Lisa picks up on this and jokingly extends it even further (l. 93). I felt a need to give a more complete explanation and I point to this practice as a means of "reinforcing your [teacher] authority" (l. 103). However, I am quick to point out that in Karl's case, and in the case of most teachers this consequence is unintended (ll. 104-106).

I wanted to be careful not to be seen as disparaging teachers, especially not my own students, while also encouraging teacher candidates to question standard practices. I position myself as one of these teachers ("those of us" l. 110) and characterize these practices as a result of comfort (ll. 110-111). I then point out that these practices may create "resistance" (l. 108) and cause "detriment to our students" (l. 113). While my intention is to avoid casting blame and so increase the likelihood that these ideas will be taken seriously, this may also serve to create a "safe" place for White teachers, which can work against meaningful, critical discussions about mathematics education by minimizing the potential negative impact of these practices (Yoon, 2012). This desire for safety reflects my own Whiteness and continuing discomfort in discussing race, even as I make race a more frequent topic of my teaching. Again I am separating intention from responsibility (Applebaum, 2010), as a means of assuaging White guilt, and possibly

sending the message that if a teacher did not intend to exercise authority then she does not need to do anything about it. By focusing on intention to create a "safe" environment I am positioning myself as a "good" teacher (a good teacher would not blame his students). However, this focus on intention can then become a justification for not engaging with social justice mathematics. While framing my original question in terms of who benefits encourages a critique of the authoritative teacher (my goal at the time), it also limits the discussion by implying that either the teacher benefits or the students do. However, there are moments when this or any particular practice may be simultaneously beneficial and detrimental to either teacher, students, or both. Considering the possibility of simultaneous benefit and detriment could create a more expansive discussion.

As we suggest that there is some detriment to the use of abstract mathematics in ways that position the teacher as authoritative, Jeff challenges this idea (below) in ways that reflect the normalization of the dominant discourses of mathematics education. He mostly removes teachers and students from the picture creating a focus on the power of discourses. However, this section also positions mathematics teachers as largely helpless to change what is taught.

115. Jeff: I mean the only trouble I'm having with that is that
116. I mean it ((the quadratic formula)) is the way to solve for zeros in [a
117. Teacher: [I'm not
118. Jeff: I know that
119. but when this was first invented you know
120. hundreds of years ago
121. so now it's being taught to very few people who are very specialized
122. but it has a life of its own
123. and so it progresses through the centuries
124. so we're still doing the same thing
125. so there's some just momentum of that way of teaching and so
126. Teacher: and that's where discourses
127. Jeff: now we have to ask a different question about
128. now this question who is benefitting

As Jeff begins he refers to the quadratic formula ("it" l. 115) as "the way to solve for zeros" (l. 116). The use of the auxiliary verb "are" and the definite article "the" position the quadratic formula as the one and only way to solve for the zeros of a quadratic function (it is not, even within dominant mathematics). This excludes the possibility of other methods and assumes the educational value of this mathematical task. In this way Jeff defends this traditional teaching practice and turns the conversation away from the focus on teachers and teacher authority. I try to break in (l. 117) in order to return the conversation to a focus on teacher authority; however, Jeff maintains his speaking turn. The striking part of this turn (ll. 118-125) is the lack of agents ("was first invented" l. 119, "being taught" l. 121), along with a granting of agency to the quadratic formula ("it has a life of its own" l. 122, "it progresses" l. 123). This highlights the powerful influence that these dominant discourses have on mathematics education. However, it also suggests that doing anything different is impossible and may justify not trying. The end result is that "we're still doing the same thing" (l. 124) as a result of the "momentum of that way of teaching" (l. 125). This, "we" (l. 124) is the only time a human agent is mentioned in this section, and Jeff has positioned them/us as helpless to teach in a different way.

Positioning teachers as helpless is another way to make the conversation "safe." As I made the conversation safe by focusing on intention earlier, Jeff has made it safe by removing possibility for change from mathematics teachers. This seems like a perfect connection to how discourses work, which I try to make (l. 126); however, Jeff has not finished his point and he continues by bringing the conversation back to the question of "who is benefitting" (l. 128). It seems that despite his challenge to the possibility of change, Jeff is demonstrating his willingness to question teacher practices. This, in part,

helps to maintain an image as a "good" student, despite his disagreement.

This transcript illustrates the difficulty of escaping the discourses that link teacher authority to teacher knowledge. We are influenced by the traditional subject positions of teachers as authoritative, which is reinforced by positioning students as recipients in the classroom and by the dominance of abstract mathematics. Our own desires for "goodness" lead us to create justification (whether of intention or agency) that undermine questioning the effects of our teaching. Further by linking teacher authority to knowledge, beginning teachers can be positioned as having authority over their students (because of their superior knowledge) and, at the same time, as helpless, because they lack the knowledge gained from teaching experience. In this way they are mathematically, but not pedagogically knowledgeable.

Taking Responsibility for Our Teaching

Students as Oppositional

Following our week 6 class the students began working on and implementing their action research projects. In our 9th week the teacher candidates were refining their plans. Because of some conflicts with student-teaching schedules I met with Stella and Jane separately from the other teacher candidates that week. As Stella, Jane, and I were discussing their projects Stella brought up the idea of telling students about her project and students purposely undermining that project. Through the discussion she constructs an oppositional relationship between students and teachers that she uses to justify the use of teacher authority. While the discussion suggests a more active position for students, by fitting it within the discourses of individual responsibility (and blame) we conclude by focusing on the teacher's responsibility to try to fix the relationship.

1. Stella: I did have one question
2. so in one of these articles
3. it said you're supposed to tell your students
4. what you're doing
5. which in class I think we talked about
6. and we said either way.
7. Teacher: I think it's a good idea
8. to be open with students
9. and to let them know
10. that you want them to do their homework more
11. and so you're going to try out some different things
12. to see if it encourages more of them
13. to do their homework kind of thing.
14. Stella: But don't you think
15. that will get in their heads
16. like they know why you're doing it
17. so they might not do it you know what I mean
18. and I also feel like
19. what if my students don't like
20. because they might not like doing quizzes
21. instead of just copying down homework for participation
22. so I'm afraid they're going to be like
23. yeah right I'm not going to do my homework.
24. Jane: Yeah. that's what I'm worried about
25. if they look at an assignment
26. and they think
27. oh she's just doing this
28. to see if I'm going to do it
29. but I don't like this
30. so I'm not going to do it
31. so she won't do it again
32. I can just like see them having that kind of mentality.

Stella begins this topic by referencing one of the articles that suggested a teacher is "supposed to tell your students" (l. 13) about action research work. By using "supposed to" Stella seems to view this as an infringement on her authority as a teacher, since she is being told what to do. I reframe this more positively, saying that it is a "good idea to be open with students" (ll. 8-9). This positions students as co-participants in their education, rather than as recipients of education. As Stella continues it appears that her main concern is that her students will undermine her efforts (ll. 16-17). This is a particular

concern when students do not like something, such as homework (l. 20). These two statements position students and teachers in important ways. First they position teachers and students in an adversarial relationship with students taking on the negative (from this perspective) role of resistance (apparently without reason) to Stella's efforts. They are positioned as resistant just because it "[got] in their heads" (l. 15) suggesting that they act without thinking; resistance becomes a reflex. While these statements position students as agents, they are portrayed as only using that agency in negative ways. This positioning is consistent in all of our discussions so far; students are only viewed as agents when they somehow resist our efforts to teach, otherwise they are passive. Further these statements suggest that what students like or "don't like" (l. 19) is not an important enough reason for them to resist her efforts. Thus student preferences are less valid than teacher preferences (or the preferences allowed by the discourses). This subordination of students' ideas comes from a dominant discourse of teacher authority.

As a result Stella/teachers are assumed to have the authority to make these decisions regardless of student desires. Jane agrees with these positions (l. 24) and suggests (as did Stella) that students may try to manipulate a situation if they do not like what is going on (ll. 27-31). This positioning of students as resisting reflexively fails to recognize the power differences in traditional teacher-student relationships. In the traditional relationship what a teacher views as "manipulation" may be an attempt at resistance for a student on the low end of a power relationship. The adversarial relationship portrayed here is used as a justification for maintaining and using teacher authority in ways that promote traditional teacher-student relationships. The justification of teacher authority in this way functions to cover up the inequities of traditional teacher-

student relationships. If students resist reflexively, then teachers cannot be expected to have reciprocal conversations with them and are justified in using authority to quash student resistance.

In the cases of Stella and Jane there may be some fear of loss of control over their own destiny. This conversation is specific to the context of their action research projects, which makes up the majority of the grade they will receive in my class. They have both been good students (as traditionally measured) and may have been afraid to give some control over their own grades to students. In this way the pressure of an external evaluation structured my relationship with them in a way that pushed them to focus on portraying themselves as "good" students. This focus puts their immediate interests at odds with the interests of their students and pushes them to reframe their students as adversaries. This discourse of control and who is in control of the classroom, is part of the dominant discourses of teacher authority.

From here the discussion begins to shift as I try to present a different perspective on teacher-student relationships. In the process, Stella and Jane adjust the ways they have positioned their students. However, the dominant discourses continue to shape our focus on who is to blame for adversarial classroom relationships.

33. Teacher: Um I don't know
34. I don't think most students
35. take that kind of oppositional attitude towards teachers.
36. Stella: Yeah.
37. Teacher: And if they do
38. that's probably a relationship you need to work on
39. most students want to try to please their teachers.
40. Stella: Ok but you know that one class
41. that's kind of
42. you've come to observe
43. and there's a few really difficult relationships
44. like that I feel like I haven't learned to deal with them yet

45. and I don't know how
46. to establish like trust with a few of those students
47. before I do this project you know
48. they'll be like mad at me for disciplining them in class
49. or one girl is literally
50. like I'm grounded because you haven't entered my grades in
51. and I'll be like I legally can't
52. I give them to Mona
53. and she knows that
54. like I tell them that
55. like I have some kids
56. like that in that class
57. who are just like set on like being opposed you know
58. so I don't know how
59. I can't fix those before doing this
60. I don't know how to fix them
61. and that's the class that struggles with homework participation the most
62. do you know what I mean
63. and there's like 17 of them so there's like a handful that
64. Teacher: Like how many
65. Stella: Well maybe like three I guess
66. Teacher: So
67. Stella: Yeah you're right
68. Teacher: That's not going to completely ruin your feedback
69. Stella: Yeah you're right ok.
70. Teacher: So.
71. Stella: So I should tell them then?
72. Teacher: Yeah I think so. You like your chair down that low?
73. Jane: Huh?
74. Teacher: You like the chair down that low.
75. Jane: Yeah I just felt like I was really high
76. yeah I think you're right
77. maybe I will tell them about it
78. Because I feel like most students
79. if it's going to go on their grade
80. and if like they usually do it
81. and they don't like the assignment
82. they're still going to do the assignment
83. because they care about their grade.
84. Teacher: Yeah.
85. Jane: Ok. I'm glad we got that figured out.

To begin shifting the discussion in a new direction I present a more positive view of students (ll. 34-35) and draw from dominant discourses in placing responsibility for that

relationship on teachers (ll. 38-39). Stella responds by pointing to a specific example from her own teaching. This time though, she restricts her comments to "a few really difficult relationships" (l. 43). While she goes through specific examples of interactions, her frequent use of negative terms in describing her own ability, stands out ("I haven't learned" l. 44, "I don't know how" l. 45, "can't" l. 51, "I don't" l. 58, "I can't" l. 59, "I don't" l. 60). This negativity is uncharacteristic of Stella's comments and positions her as helpless within the context of these particular relationships. This can be a way of avoiding the blame that I have suggested. However, as we continue talking she notes that there are really only three students (l. 65) and decides that she should talk to them about her project. Jane adds on that "most students" (l. 78) will generally work with her. While their closing comments are not developed enough to show a different subject position for students, Jane and Stella do demonstrate a greater willingness to work with students. Our focus here and previously on making teachers responsible for what happens in the classroom, drawing from dominant discourses of authority and individual responsibility, led us to regularly discuss who is ultimately responsible (i.e., to blame). This blame game effectively distracted us from developing an understanding of teacher and students in reciprocal, productive relationships.

Teacher and Students as Doers

As we moved into the 11th week of classes the teacher candidates had begun implementing their action research projects and reported on them during that class. Jeff had set himself an ambitious task to dramatically increase the amount and quality of group work in his class (this was a big change from how his mentor teacher taught). He had put a lot of effort into it and felt that his students did not appreciate what he had

done. As a result in the transcript that follows, he positions himself as a hard-working teacher and students as ungrateful for the work he is doing. This discourse (hard working teacher/ungrateful students) is common in education and grows out of the dominant discourse of responsibility. The teacher (through hard work) has done his job (i.e., acted responsibly), but the students have not met expectations (i.e., acted responsibly). In this case, this discourse serves to exonerate the teacher for a lesson that did not go well and as a potential justification to reduce efforts to make learning mathematics more meaningful to students.

((In this transcript *italicized text* indicates reading))
1. Jeff: So I had four real days to do lessons
2. and I had three lesson plans to work with
3. and so my whole idea is to try
4. and get conceptual development
5. and um concept retention is really
6. because I really feel like my kids are
7. I mean I talk about the same things over and over again
8. and they don't remember what I'm saying they don't
9. the algorithms they don't remember any of that stuff
10. so I'm trying to figure out ways
11. to get them to remember what I think is more important
12. like how to solve problems just as a general thing

In this opening section Jeff sets up the original problem that he is trying to address which focuses on "conceptual development" (l. 4) and "concept retention" (l. 5). This language points to the discourse of constructivism and serves to position him as a "good," responsible teacher. He notes that he talks about things "over and over again" (l. 7), but that students "don't remember" (l. 8). To this point he explains what happened in neutral terms. Students are not blamed for not remembering and he is "trying to figure out ways" (l. 10). In this way he is taking the problem, as he sees it, and looking at his own teaching as part of the problem. In some ways this portrays a sense of responsibility to his

students. However, we can begin to see the ways in which he positions himself/teachers in relation to his students in ways that focus on his actions. Throughout this section he is the agent (note the frequent use of I) and students are mostly passive recipients of action. He is the one "trying" (l. 10) to make improvements. Further his improvements are based on what *he* feels (l. 6) and what *he* "think[s] is more important" (l. 11). Dominant discourses of individual responsibility seem to compel Jeff to try to prove his "goodness." This creates a focus on his efforts and deflects attention away from what is happening in his class.

As he continues he introduces one of the lessons that he taught to his students. This lesson was to involve students in analyzing some data about a sports car. He introduced the lesson by showing students a brief video about the car, and he also showed the video to us.

13. Jeff: so um what I did
14. so let me give you an example of one of these lessons
15. I have to turn this on
16. ok so I like cars.
17. Gavin: Is that a Henessy Venom?
18. Jeff: Yep this is a venom
19. they had a world record breaking run
20. so let me show you what I did for them
21. because I wanted to set the stage for this lesson
22. so I'm going to have to well ((plays video))
23. so yeah so before I did this whole exercise
24. was because I figured kids like cool things
25. and I thought this is a cool
26. this just happened not that long ago
27. it was all over the internet
28. Lisa: cool
29. Jeff: and it went really fast
30. and I thought I could put this into my lesson plan
31. and I don't know so here's a spec on the next slide
32. I've got to go back to it
33. ok so this was the car that raced to the top
34. and this is how I compare some pretty big name cars

35. you know cars Gavin what do you think of that?
36. Gavin: What do I think of what?
37. Jeff: So the McClaren P1 in 9.9
38. so guys a quarter mile in 9.9 seconds
39. and going 163 miles an hour
40. and that to me just boggles my mind

Throughout this section Jeff focuses on "what [he] did" (ll. 13, 20). Notice again the frequent use of "I" with an action verb. The focus in this section is entirely on the information that Jeff gathered about cars and what he thinks about it. Students are only mentioned twice. First, they are the people he is doing this work for (l. 20). Then they are young people who like "cool things" (l. 24), although Jeff is the one deciding what is cool and what is not. Students are not consulted as to what they think or what they would like to learn. This positions student perspectives about the class as irrelevant and again they are positioned (largely by their absence) as passive recipients of teaching. These positions, with teacher as active and students as passive, maintain and justify the authority of the teacher in ways that marginalize students.

Now Jeff begins to get into the actual lesson. Here students come more into the picture. However, they serve to illustrate how much Jeff's hard work has been in vain.

41. Jeff: so I'm thinking this is going to be cool
42. they're going to love this
43. and that was my thought process going through this whole lesson plan
44. maybe not so much and seriously
45. and this is the whole point
46. I'm working hard trying to come up with a lesson plan
Lines 47-62 ((Jeff reads a portion of the worksheet he created for his students))
63. so there's this equation up here
64. and this is just the standard acceleration equation from physics
65. you don't have to know that it's from physics
66. but you know they know what distance is
67. they know what time is
68. and I'm explaining d is distance g is the number of gs
69. so 16 so in units of feet
70. so when gravity is 32.2 feet per second squared

71. and the formula is one half g
72. so that's what the 16.1 is
73. I didn't explain that to them
74. but you guys all understand that all right
75. so I'm just giving the preliminary information
76. I'm trying to get them to see
77. ok is this really a fast car
78. is this a cool car or what
79. I'm trying to get them to really take to this exercise right
80. they were so confused all right
81. so I said *write an equation*
82. whose solution is the time it takes the car
83. to travel one fourth of a mile
84. so using this equation just write an equation
85. so all they had to do was put 1320 in for d and 1 in for g.

While Jeff thinks that "this is going to be cool" (l. 41) and that students are "going to love this" (l. 42), they do not (l. 44). This happens even though Jeff is "working hard" (l. 46). Thus Jeff positions himself as working hard to try to do something that his students will love, but they do not. This positions him/teachers as hard-working, but underappreciated, while students are ungrateful, since they do not appreciate the results of this hard work. Jeff positions himself as a "good" teacher despite the lack of engagement of his students with the lesson. As he explains what comes next he minimizes the work expected of students ("just the standard" l. 64) and praises the students' knowledge ("they know what distance is" l. 66, "they know what time is" l. 67). Despite the students' knowledge and the simplicity of the task, when he explains (l. 68) and tries (l. 76) and tries (l. 79) the students "were so confused" (l. 80). He continues to minimize what was expected of his students, noting that they were to "just write an equation" (l. 84) and "all they had to do" (l. 85) both comments make it seem that very little was being asked of the students. By minimizing what he was asking students to do and pointing out what they already know he positions himself as having done everything he should to prepare and teach his

students. He is in this way helpless to reach his students since he has done everything he knows to make this lesson relevant to his students. The rest of Jeff's report continues in a similar fashion, with Jeff emphasizing how much work he is doing (he goes so far as to say "I'm killing myself" at one point), while relatively little is being asked of his students.

Throughout Jeff's report and the following question and answer period no one offers a significantly different positioning for teacher or students until Jane makes this comment below.

495. Jane: My question is on like
496. what kind of things do you give them to do for group work
497. because I think that depends on how much they're willing to work together
498. because sometimes if it's just like they're doing algorithms
499. some kids get it some kids don't
500. so obviously they don't want to work with each other
501. but if it's something where it's going to benefit them
502. to work with their group
503. they're more likely to engage in group work
504. Jeff: Um what do you think of when you say will benefit them
505. what do you think?
506. do you have something specific in mind when you say that?
507. Jane: Like I don't know
508. like a couple days ago
509. we do a lot of group work
510. and we
511. they had to make scale drawings of seven islands
512. and so within their groups
513. they all had to work together
514. and check each others'
515. and so some of them actually did more than others
516. but they were all doing the same thing
517. and all checking each others'
518. so then they still had to work together
519. I don't know
520. Jeff: So but ok it sounds like you have an activity that's well-designed
521. and that's something that I'm still struggling to come up with
522. and but when you look at what's out there for secondary III
523. I feel like I'm having to wrestle all that material into something
524. I'm not very good at yet

525. it's just that's the sustainability thing I'm talking about
526. I'm trying to figure out how to take the babysteps that Gavin is talking about

The crux of Jane's comment is that students will work well in groups when "it's going to benefit them" (l. 501). In contrast to Jeff's comment there is very little mention of what the teacher does and most of the comments are about the students; what they "do" (l. 496), their willingness (l. 497), what they want (l. 500), and their engagement. She implies that to be successful these are the things that a teacher needs to consider. Thus, rather than focusing on how much work there is for the teacher, the teacher can view the students as worth listening to and responding to the messages they are sending (Margonis, forthcoming), in this case through their participation or nonparticipation. Jane gives a brief example of something that students did in her class (note again the focus on students rather than on the teacher). Jane's comments consistently position students as making choices and as participating in their education. These choices are not made in order to be oppositional to teachers (in contrast to her comments in week 9), but based on whether the students see an activity as worth doing.

As Jeff responds he recognizes part of the value of Jane's example, since he notes that the activity is "well-designed" (l. 520). However, he continues to position the teacher as the one doing all of the work ("I'm still struggling" l. 521; "I'm having to wrestle" l. 523). Jane's attempt to reposition teacher and student includes a different role for the teacher and an increased role for the student, with an emphasis on what will be beneficial to the student. While the teacher still seems to be the one deciding what is beneficial to the student, these positionings are more in line with, even if insufficient, for an equity focus in the classroom.

Part of the difference in the ways Jane and Jeff have positioned students and

teachers can be explained through the different contexts in which they were student-teaching. Jane worked in a science and mathematics focused charter school (middle-school) with a teacher who was a master at creating real-world group projects, which she used daily. After 7 months working in this context Jane was quite comfortable developing group projects. In contrast, Jeff was working in a traditional public high school with a teacher who occasionally did group work and rarely made real-world connections. Jeff's report represents one of his first attempts to create this kind of lesson and he was clearly frustrated. These local contexts clearly affected the discourses that they drew on.

The Teacher Makes Mistakes

In this section Lisa reports on the progress she is making on her project. As she does she is very careful to ascribe the problems that she is facing to her choices and the things that she is trying to do in her classroom. Interestingly this positions her as the main authority in her class. Everything bad that happens in the class happens as a result of her actions; this positions her as solely responsible for what happens in the class and seems to be an example of combining dominant discourses of individual (teacher) responsibility with avoiding deficit thinking. This is one of the few instances where the discourse of responsibility is consistently used to place responsibility on the teacher. However, this combination does not leave room for active student subject positions.

Lines 1-6 ((Lisa asks her classmates to take notes for her))
7. ok so I'm first going to talk for minute about the reasons why
8. I started this whole idea in the first place
9. um these are
10. that I had no idea what my students were learning at all
Lines 11-26 ((Lisa explains some of what led up to this project))
27. and then I started having problems
28. so I realized I didn't know what they knew
29. and then I started to plan lessons anyway

30. and this like divide grew in my class
31. and you should see the scores for their final
32. it's like half As half failing
33. they're like there's like nobody in the middle
34. it's like crazy
35. so I think like people were getting it
36. and I didn't really pay attention to them
37. because I knew that people were struggling
38. and we kept like repeating stuff
39. and people got bored
40. and then I realized that that was happening
41. so I switched my focus
42. and started paying attention to them
43. but by that point people had given up on trigonometry in general
44. and the kids that were sort of lost
45. like still were and it just became like this huge thing.

Lines 46-53 ((I interject a question about classroom management))

Notice throughout how frequently "I" is used as the subject of the sentence. This positions her as the agent in each of these statements. While most of these statements refer to some lack or mistake on her part, by focusing on Lisa as agent they still ascribe power only to her. The power of her decisions can create major problems in the classroom. When students are brought up it is usually in a passive sense with ("people were getting it" l. 35 or "people got bored" l. 39) or without an agent ("divide grew in my class" l. 30). There is little agency shown on the side of the students, which positions them as powerless in order to make them blameless for the grade disparity and other issues that were in the class. This appears to be an example of "being in control" (Gutiérrez, 2009) in which Gutiérrez recommends that mathematics teachers take responsibility for what happens in their classroom. However, Gutiérrez places this in tension with "not being in control" which recognizes that students are also participants in their education and they have a history in their community and their school. As a result students may make choices not to participate for a variety of reasons. Additionally, there

is no mention of systemic factors (prior mathematical preparation) that influence students' ability to succeed. By positioning herself as the sole agent Lisa disallows students' participation in their own education and does not acknowledge systemic factors that have set up her class and her students for these interactions.

Consistently throughout the semester we, like Lisa, drew on the dominant discourses of individual responsibility and accompanying discourses of teacher authority as we discussed our efforts to teach mathematics, especially mathematics for social justice. Frequently, these discourses led us to focus on who was to blame. Most of the time we used these discourses to deflect blame from teachers placing it alternately on school administrators, policy makers, vague "theys," or uncooperative or ungrateful students. I occasionally suggested that teachers were partly to blame and Lisa took blame wholly on herself. When teachers accept responsibility (or blame) it can feel like progress towards more socially just teaching and my initial inclination was to present these discussion in that light. However, these discourses do not allow for a meaningful role for students in their education nor do they create a way for teachers to use authority in more socially just ways. Thus discourses of individual responsibility, even when used by teachers to hold themselves accountable, restrict our efforts to position teachers in ways that promote socially just mathematics teaching. However, there were moments when we discussed more respectful participatory subject positions for our students and repositioned teachers in response. Our attempts to position students and teachers in nondominant ways are the focus of the following section.

Teacher Authority and Responsibility to Students

Students as Participants: Talking With Them

Our class in week 7 was focused on developing the teacher candidates' action research questions. They had come to the class prepared with ideas that they wanted to try out. Each of the teacher candidates had a chance to share their ideas; we asked each other questions and discussed our ideas. I gave them some time to work on developing a plan for their projects and we planned how to write their project ideas into research questions. In this portion of the discussion I briefly addressed framing questions in ways that avoid deficit thinking about students. Following my comments, Esperanza picks up the discussion to ask about talking to students about her action research project. By asking this question Esperanza repositions students as people whose thoughts and desires are important, while maintaining the primacy of the teacher's needs.

Lines 1-28 ((I begin a discussion about how their research questions frame students))
29. Esperanza: I have a question.
30. Teacher: Yes.
31. Esperanza: Would it be a good idea to let your students know
32. what you're looking forward to accomplish or whatever
33. to have them be aware
34. to have input somewhat
35. in what they think it would be better to do
36. in terms of
37. oh I don't like this
38. I think we should change this
39. is this even a conversation that you could have in your class um
40. Teacher: Um . . yes
41. depending on the topic
42. so for example
43. if you want to make your math lessons more relevant to students
44. you've got to talk to them
45. because the only way you're going to find out
46. what's relevant to them
47. is by talking to them.

Even though Esperanza is suggesting that we talk to our students, she places limits on the

value of their contributions (“be aware” l. 33, “input somewhat” l. 34, and simple choices ll. 37-38). The importance of student participation is recognized, but only in limited ways. While this positioning of her students may be necessary to maintain an image of a “good” (in control) teacher, this seems to be an attempt to fit a more active student position in the dominant discourse of teacher authority. As she continues, though, she broadens the range of student participation to include more open participation (ll. 35-38). As I respond I suggest that “you’ve got to talk to them” (l. 44), making it an imperative that she talk to her students. However, in my justification I refocus on the need of the teacher candidate to gather information (l. 45) rather than on the needs of the student.

My suggestion would have relied more on student input than what Esperanza intended and in her response she gives an example of the kind of discussion she intended. As she does so she creates a kind of role play of the kind of discussion that she is envisioning. The students give opinions, again in limited ways, and Esperanza portrays herself as authoritative.

48. Esperanza: Oh no no I don't mean
49. like asking them once I have set up my question
50. but about for example
51. I really like the idea of Connie that Stella and Jane were talking about
52. because I have issues with my students turning in homework at all
53. and either this or whatever
54. like even talking about it
55. do you guys look forward into having different types of homework
56. Because they always complain about oh you give us homework everyday
57. I'm tired of it ((imitating students))
58. but you don't do either one ((imitating self as teacher))
59. so it doesn't really matter
60. if I give it to you or not
61. and I don't know why you're complaining
62. but the thing is like
63. well if I were to give you less problems
64. but more challenging problems
65. would you like that?

66. Like that kind of discussion?
67. Do you think that is ok to have with your students?
68. and based your own project
69. on what they think
70. for example if I like oh
71. I want to make it relevant and students are all
72. oh I don't care if you make it relevant to me whatsoever
73. even if you try. ((imitating students))
74. Like that's probably not a good project to me
75. to even do if my students are going to act that way
76. because I mean some of them are very vocal
77. at least I have some
78. I'd like to do something
79. that is not going to affect in a bad way
80. is that what I mean
81. like am I making sense so that
82. Because either way it's going to affect them
83. if it's only me doing it
84. and quietly thinking about it
85. um so is that a good way to go about asking them?
86. Teacher: Um yes
87. Esperanza: Ok.

To begin the focus is on the "issues" (l. 52) that Esperanza/teachers have with students turning in homework. There is no mention of benefit or detriment to the students, placing the emphasis again on the teacher's needs. She points out that her students complain about homework (l. 56) and mimics their complaints and her responses. This imitated dialogue represents students' views, but positions students as whiny and disengaged from school (ll. 56-57), without any critique of why they might feel that way or whether that view is representative of all students. In response, Esperanza positions herself as an exasperated teacher who is tired of her students' excuses. She portrays herself as out of options ("you don't do either one" l. 58, "it doesn't really matter" l. 59). Similar to Jeff's comments above, this is part of an image of teachers as hard-working, but unable to overcome the challenges that are presented by today's youth and their attitudes about school and the lack of respect they have for authority. To this point she is drawing on the

dominant discourses of teacher authority. However, she then suggests that she could approach her students with a question about what they would like (ll. 63-65) and then base her project on her students' responses (ll. 68-69) This suggestion recognizes the importance of student thinking and listening to them and suggests the possibility of shared authority (rather than either/or). She then critiques a traditional authoritative position for teachers who make decisions without gathering input from students (ll. 70-73). Her statement in line 74, that it is "probably not a good project" if students do not support it, is an important one. This begins to position teachers as responsible to their students, and is only possible with her repositioning of students and their perspectives as important. Her desire is that she not "affect [her students] in a bad way" (l. 79) and to avoid "only me doing it" (l. 83). While in many ways this is still about her and her project she also recognizes the value of her students' thinking and desires.

As Esperanza shifted the focus of our discussion to what was happening in her classroom she brought students' perspectives back into our discussion. While these were first portrayed in negative and limited ways, Esperanza proposed a change in her teaching that was more respectful of and valued students' perspectives about their learning and their experiences in class. Her needs and desires were still central; however, her students' needs and desires take on a more prominent role than in previous discussions. While not fully worked out, these changes suggest the possibility of a relationship of shared authority between teacher and students rather than the binary relationship of control/out-of-control.

Positioning Teachers and Students to Address Social Justice Topics

In preparation for our week 12 class the teacher candidates read an article by Gutstein (2012) in which he discusses how one of the high school classes he teaches addresses various social justice topics including the spread of HIV/AIDS in a local community. In the beginning of the class Karl brought up the question of how to bring up difficult topics, such as HIV/AIDS, and have students take them as seriously as they should. Later in the class I direct the teacher candidates back to Karl's question. In our responses to this question we describe the types of student responses that we fear, our own limitations, and what kind of relationship we hope to have between students and teachers.

1. Teacher: So let's talk about difficult topics
2. how you address difficult topics like HIV and AIDS
3. um and any number of others
4. and I think this will help us move into some of these discussions as well

Lines 5-61 ((we discuss self-citation, decide to put off a discussion of Gutstein's (2006) concept of political relationships, and I remind them of Stella, Lisa, Esperanza, and Jane's lesson demo that they had seen the previous weekend))

62. Teacher: So let's let's talk about this
63. so somebody there ((at the lesson presentation)) brought up the idea
64. that if you're teaching this lesson on the west side
65. that it could be possibly discouraging to students.
66. How would you address that?
67. Gavin: Well just that life sucks
68. and that's how it is.
69. Multiple: ((laughter))
70. Jeff: Isn't it all in the context of being able to create change
71. if you can do that
72. it doesn't have to be inherently negative
73. being reminded of reality
74. and if that's all you're doing is saying yeah tell me something I don't know
75. Teacher: Isn't well ok I want to hear you out
76. is there any benefit to doing that?
77. Jane: Doing what?
78. Jeff: Well maybe I mean to let them know that you're aware

79. Teacher: To remind them mhmmm why might that be beneficial?
80. Jeff: Because they may feel that you aren't
81. and they can't approach you and talk about it
82. Teacher: K.
83. Esperanza: True.
84. Lisa: True.

From the beginning I position the teacher as the authority who is bringing difficult topics to the class ("how you address" l. 2). While this is what Karl's question was about, I do not challenge this assumption. This leads us to continue the conversation in a way that assumes the authority of the teacher. In the skipped portion of this transcript I ask the teacher candidates to recall the lesson presentation that Esperanza, Stella, Lisa, and Jane had done the weekend prior. I wanted them to address these ideas from a specific context rather than in generalities. In this presentation they described a lesson they had developed that demonstrates the unequal distribution of parks in our city based on geography and population density (this corresponds to income and race as well, although this was not discussed in their presentation).

During the presentation one of the White teachers in attendance brought up the point that she would not do a lesson like that with her "west side" students (mostly low-income students of color), since they already know that they are treated unequally and this lesson could further discourage them. As I bring us back to the topic of Karl's question I reference this comment (ll. 64-65). I then narrow the question to focus specifically on those topics that "could be possibly discouraging to students" (l. 65) and ask the teacher candidates how they would address (again positioning them as agents) this situation (l. 66). I wanted the teacher candidates to take responsibility for their teaching and to see themselves as capable of teaching mathematics for social justice. This is part of why I position them as agents in this section. However, as with other times

when we draw on these dominant discourses, I have not left room for meaningful student participation.

After Gavin's joke Jeff points to the importance of "being able to create change" (l. 70). The passive construction here leaves it unclear as to who is creating change. However, if it is parallel to the structurally similar "being reminded of reality" where it is more clear that the students are the ones "being reminded," we might assume that students are also the ones enabled by the teacher to create change. This passive role for the student maintains the authority of the teacher, even if that teacher intends to "create change" (l. 70). I start to tell students what I think ("isn't" l. 75), but decide instead to see where the conversation will go. Picking up on Jeff's point I ask if there is benefit to just pointing out inequity (ll. 75-76, 79). Jeff mentions a benefit to the teacher-student relationship in overcoming some of the teacher's authority to make her/him more approachable (l. 81), which other students agree with (ll. 83-84). This point suggests that students are careful in their choice of relationships, positioning them as participants and careful as they negotiate the school system. At this point this improved relationship between teacher and students should benefit the students as they will be more likely to have someone that they can work with and who will listen to them.

I continue asking questions to see where the students will go with these ideas. As Lisa responds she seems to suggest that bringing up difficult, social justice topics is beneficial because it can increase teacher authority.

85. Teacher: And why would they care if you as their teacher are aware
86. Jeff: Because you have authority
87. that you're trying to hand out to them.
88. Teacher: K. other thoughts on that?
89. Lisa: Yeah. Because you're trying to teach them stuff
90. if they think you have no idea what's going on

91. and they don't get you at all
92. they're not going to like participate in your classroom
93. Jeff: So you can't create relationships you mean
94. Lisa: Yeah. So I think that's something
95. that I never really thought of that much
96. making them aware that you're aware of it
97. Teacher: K. Other thoughts on that?
98. Lisa: Um on the whole question or
99. Teacher: On this specific part.
100. Lisa: I don't know what this specific part is
101. Teacher: Um is there benefit in just making students aware of social issues
102. like this like that the parks are divided unequally in this city.
103. Lisa: Um I think that you have to talk about the actual thing
104. before you can start to talk about why that thing is occurring
105. and I don't think that that is a given
106. that all students know reasons
107. that could potentially explain why things are unfair.

Lisa highlights the importance of what students think of their teacher. If their opinion of the teacher is low (in terms of intelligence and understanding) then "they're not going to . . . participate in your classroom" (l. 92). This is a very similar statement to one Jeff made in week 4, "if they [students] don't think you have the knowledge you can't teach them in any way." Both Jeff and Lisa are relating a teacher's knowledge and demonstrating that knowledge to students, with students' willingness to listen to that teacher. The implication is that a lack of teacher knowledge (or not communicating knowledge to students) may result in students not participating in class.

If we are talking about teaching mathematics in a traditional sense (as Jeff was) then the knowledge/authority link is restricted to mathematical knowledge. However, if we are talking about a social justice context (as Lisa is) then that knowledge is expanded to include sociopolitical knowledge. This is a disruptive view suggesting that being a mathematics teacher requires sociopolitical knowledge in addition to knowledge of mathematics. However, increased knowledge is still linked to increased authority. There

is no discussion of benefit to the students from this kind of teaching. While the benefit may be assumed, it seems that our primary concern is how bringing up difficult topics in the classroom will affect the teacher. As I rephrase the question (l. 101) I maintain this focus on the teacher as agent who is "making students aware," which positions students as mostly unaware of social issues. Lisa also notes that we should not assume that "all students know" (l. 106). Thus as we try to position ourselves as taking responsibility for teaching mathematics for social justice (using dominant discourses of responsibility) we are reaffirming a central focus on the teacher as classroom authority.

As we continue Gavin points out the importance of discussing racism with students. In response I ask the rest of the teacher candidates to consider what it means for them (most of them) as White teachers to discuss racism with students of color. Even though Gavin brought the topic up Esperanza, the only Latina teacher candidate, is willing to respond, at first.

Lines 108-126 ((Gavin brings up the importance of pointing out racism as a relevant topic, although he does not connect this to either students or teachers))
127. Teacher: So what might it mean for most of you as White teachers
128. for your students who are African American or Latino or White students
129. as well for you as a White teacher to acknowledge racism does happen
130. and it happens here
131. ((some untranscribed discussion about the heater, which was making loud sounds))
132. Teacher: So my question was
133. what might it mean to a minority student
134. or even to a White student
135. for a White teacher to acknowledge racism?
136. Esperanza: I want to say something
137. on my way back from Logan
138. me and Annie were talking
139. and we were talking about role models and stuff
140. and she said um she answered your question
141. she said it makes students aware
142. that we as White teachers know we're allies
143. and we're part of their community
144. even though we might not live on the west side

145. but I've been there
146. I grew up within that neighborhood within that school
147. and it just helps them see that
148. as part of the majority
149. I'm an ally to them
150. I'm that connection to bring them over here
151. that's what she said

My question is originally directed to the White teacher candidates (l. 127). It is interesting that the first student to respond is Esperanza. Possibly the other teacher candidates are less comfortable explicitly discussing race. Since the question was not directly to her, Esperanza begins with "I want to say something" (l. 136), almost asking for permission to respond to the question. This allows her to position herself to answer the question without appearing to become part of the group of White teachers. However, in answering the question she references a comment made by Annie, another White mathematics teacher all of these teacher candidates know.

While Esperanza may be more comfortable talking about race than the other teacher candidates she uses the words of another White teacher to do so. She then names teachers as "allies" (l. 142) to their students as well as community members with their students (l. 143). In her comment about community ("we're part of their community" l. 143) the pronouns discursively maintain a distinction between teachers (White, in this case) and their students of color. Even as she is positioning teachers as allies and community members the distinction between the two groups is reinforced. The teacher again is the agent to "bring them over here" (l. 150). This implies that the community of color needs the connections that the White teacher has in order to overcome deficits. This perspective grows out of the dominant discourses of teacher authority within schools. According to these discourses, within the school the teacher (not students, parents, or

communities) has authority. Esperanza feels strongly about social justice work and this may be her way of trying to help her White colleagues see a role for themselves in working with students of color. The role of ally may be a way to reimagine the positioning of teachers, however, this role is not explored here.

To this point we have focused mostly on what we perceived as positive aspects of bringing social justice topics into the classroom, while focusing only on awareness (as opposed to change) and only on how these topics help the teachers. Lisa now asks us to consider some potentially negative aspects of doing so. In some ways Lisa's comments are a counter to the idea of White teacher as ally. While she does not mention race explicitly she mentions that students (probably students of color) may be right not to trust (White) teachers. This is one of the first times in this discussion that a student perspective is taken up and students are positioned as intelligent and capable. This change in perspective begins a shift in how we position students and teachers.

Lines 152-180 ((we discuss how to overcome students' of color negative perceptions about the racial beliefs of their White teachers))
181. Teacher: Lisa
182. Lisa: I feel like we need to talk about
183. some potentially negative consequences that could happen.
184. Teacher: Ok. Good.
185. Lisa: Because all I want to say
186. when you ask that question is well
187. they'll probably think that we understand them better
188. and that we're on their side
189. and all these positive things
190. but there has to be a negative side
191. Teacher: Is there risk in doing that
192. Lisa: Yeah and maybe they're
193. if they don't trust they're right to
194. so definitely negative feelings to be had
Lines 195-203 ((Jane brings up one of the articles and we figure out which one she is talking about))
Lines 204-221 ((we discuss laws regarding undocumented students and in-state tuition))
222. Jane: So my question I guess is

223. yeah well sort of
224. so it was kind of like depressing
225. I feel like when they said like
226. well hey you have to pay way more to go to school in this state
227. because you're undocumented
228. but then I feel like it's kind of like all those
229. all the other students who are like legal or whatever
230. their tax dollars are going towards the state
231. and that's the reason they get in-state tuition or whatever
232. I don't know I feel like that
233. that thought would be absolutely depressing
234. more so than the parks thing
235. I feel like the thought of paying
236. Lisa: Oh yeah
237. Jane: like $60,000 more
238. when you already don't have very much money
239. and like with that one especially
240. I couldn't see any solution or any benefit of telling
241. it was just kind of like really discouraging
242. just like ok it looks like you're not going to college

Lisa begins by summarizing many of the positives that have been discussed. She states that students "think that we understand them better and that we're on their side" (ll. 187-188). In both of these cases the primary benefits seem to go to the teacher. From here she flips the perspective to say "if they [students] don't trust [their teachers] they're right to" (l. 193). This positions students as intelligent and selective in whom they choose to trust. This also suggests that teachers may not always be worthy of their trust. These are both important shifts in how we position teachers and students and serve to momentarily disrupt the dominant discourse of (White) teacher authority and passive students. While she does not explicitly mention race, within the immediate context of the discussion the implication is that we are talking about students of color and White teachers. Positioning students as intelligent and careful navigators of a racist school system is a necessary condition in order to work *with* students and to establish teachers as responsible to students.

In response to Lisa's comments Jane brings up an example from one of the readings. This example focuses on a lesson about the costs of college including the costs for undocumented students. Jane characterizes this lesson as "depressing" (l. 224) for students. She implies in this way that teachers may be somewhat unaware of what students think and feel. As a daughter of immigrants Jane may feel a particular connection to this particular example. She does not specify a subject position for undocumented students. However, she references the conservative argument about "legal" (l. 229) students whose "tax dollars" (l. 230) earn them the benefit of "in-state tuition" (l. 231). The unstated part of this argument is that the undocumented students do not pay taxes and are undeserving of an in-state tuition benefit. This is then also connected to images of people of color who live off the work of (White) citizens through welfare and other government benefits. Using this argument Jane (intentionally or not) suggests that undocumented students are potentially undeserving of further education.

Positioning students in this way requires that teachers determine what information students have access to (this is stated more explicitly later) and which students need/deserve information. This need to determine is a specific manifestation of authority, it is an authority to judge, which is a key aspect of Whiteness (Frye, 1992), based on the implied lack of (racial) bias that Whites possess and the White certainty of right and wrong. Jane is one of the first to consider this discussion in terms of the effects on students and the potential negative effects on students. However, by considering student effects from the position of judge she creates an authoritative position for teachers in which teachers make significant life choices for students. In this way, teachers may inhibit college preparation, especially for students of color, by not providing accurate

information about college.

Following Jane's argument other students comment on the issue of the cost of college for undocumented students. As they do so they present other subject positions for both teachers and students.

243. Karl: Yeah or yeah it was really discouraging
244. um I guess in a way it could be in a way motivating
245. to think about or at least at least they realize how much it would cost
246. and it would be tough
247. but that there are probably things
248. they could do I guess.
249. Jane: Mhmmm
250. Esperanza: A way to follow up would be
251. like as a teacher investigate scholarships
252. and stuff that they can apply to
253. Karl: Right
254. Jane: But I feel like that's another problem is
255. that they don't usually offer that many scholarships to undocumented students
256. Teacher: Well there are scholarships specifically for undocumented students
257. but there are less
258. another thing I would say to do would be letter writing
259. you know for the dream act
260. um which is an act to make this a nationwide law
261. what's in Utah
262. as well as a pathway to citizenship after graduating from a U.S. college
263. so there's things like that as well

Karl's opening comment serves to position himself as not disagreeing with Jane. However, the rest of his statement begins to create a more nuanced position for undocumented students. He suggests that knowing the cost of college could be "motivating" (l. 244). This suggests that students have a resilience that can aid them in overcoming these kinds of challenges. He then adds that "they realize" (l. 245), which implies that these students are capable of comprehending this information and using it ("things they could do" ll. 247-248). In this way he positions undocumented students as capable of overcoming adversity and as capable of making their own educational

decisions. Esperanza then points out that teachers could "follow up" (l. 250). Following up assumes that the teacher has taught the lesson in question. For her teaching the lesson and sharing the information is an assumed part of being a teacher. This different framing of teacher responsibility only comes as we position students as intelligent and capable.

Esperanza's assumption and Karl's positioning of students suggest that teachers have a responsibility to share this information. She suggests that "a teacher investigate scholarships" (l. 251) and so positions teachers as responsible *to* their students to do this kind of work: work that may typically be framed as outside the role of a teacher. This is also part of teacher authority. However, in this case instead of judging, the teacher uses authority to provide information and resources to students. This reflects a positioning of teachers as responsible to their students, which differs from the dominant discourses of teachers as responsible *for* their students. The positioning of teachers as responsible to is facilitated in part by a positioning of students as intelligent and capable. Jane then points out the relative lack of scholarships for undocumented students (l. 255). In contrast to her previous comments, characterizing the lack of scholarships as a "problem" (l. 254) appears to critique the system and repositions undocumented students as innocent of the situation they are in. I also offer suggestions for what teachers can do (ll. 256, 258), positioning teachers as able and responsible to do something in this situation.

I then refocus the discussion on part of Karl's comment about students' need to know the truth. As the teacher candidates respond we go more into depth about how we see the role and responsibility of teachers.

264. Teacher: On Karl's comment is it
265. ok in this state that she's talking about
266. this is the reality the current reality right at the time that she taught that
267. do students need to know the reality

268. or should they be left to wonder?
269. Esperanza: They should know the reality.
270. Jane: But I think the question is
271. should you be the one to tell them the reality.
272. Lisa: Well what if nobody else tells them?
273. Gavin: Um that kind of brings up
274. what I was going to say early
275. I took this Chicano experience class for diversity
276. and we read uh a book I don't remember the authors name
277. but he was Mexican and came here
278. and this was one of their negative things
279. whenever anybody in authority position
280. basically White
281. was trying to help him achieve something it pissed him off
282. Because he's like I'm a capable human being
283. I can actually do things without their help
284. so that's one of the negative things is like
285. do we are we the ones that bring it up
286. because maybe it will have that kind of impact
287. that like White like I'm a capable human being I don't need your help
288. but who knows how many students are like that
289. compared to those that are like you know
290. we were talking about earlier
291. that maybe it would benefit them having an ally that's in the majority.
292. Karl: I've had a student say that exact same thing in my class
293. that he feels dumb because people try to help him
294. Teacher: Yeah and there are certainly ways to be an ally
295. or to offer help
296. that are more condescending than actually helpful
297. and that's something to think about

I set up the original question (ll. 264-268) in a way that there is really only one answer, making it difficult for the teacher candidates to disagree with me without a direct challenge. While the teacher candidates agree that students need to know the reality, Jane questions whether teachers should be the ones to tell them (l. 271). This appears to be an attempt to narrow what it means to be a teacher, drawing on dominant discourses, by suggesting that these kinds of discussions are outside of the role of a mathematics teacher. Lisa challenges this idea by implying that "if nobody else tells them" (l. 272) then teachers have the responsibility to do so. These differing perspectives seem to be

connected to positions that Lisa and Jane have laid out earlier. Jane, who positioned teachers as judges earlier, tries to restrict teacher roles to those where the teacher can maintain authority. Lisa, who positioned students as capable and intelligent, is less concerned about teacher authority than about what students need to know.

Gavin then brings up a lengthy example of how White authority figures' attempts to help may be offensive to students of color. While this appears to recognize the capability (note the repeated use of "capable") of and differences among students of color, he uses this to question "are we [as White teachers] the ones that bring it up" (l. 284). In this light, then this example can be used as a way to excuse White teachers from attempting social justice work, because it may be offensive to (male, in this case) students of color. White teachers are thus made helpless by the potential for anger from students of color. Karl provides a brief, similar example from his own teaching. In this way the dominant discourses around teacher authority/helplessness have been brought back into the discussion. Then, rather than directly challenge his statement, I reposition the teacher in this example by setting up a contrast between an actual "ally" (l. 293) and being "condescending" (l. 295). This returns responsibility to the (White) teacher, rather than use the potential anger of students of color as an excuse for inaction. In this way I continue to center authority in the teacher.

At this point Jeff relates an experience that, on the surface, appears to be unrelated to our discussion. However, he uses this story to make a point about teachers and reinforce the positioning that Gavin has explained.

298. Jeff: So I have a niece
299. she's from Argentina or her mom is from Argentina
300. so she's half Latino
301. she was told if you apply for this scholarship

302. her dad works at Iowa State
303. and she wanted to go there
304. but the tuition is quite high even for in-state
305. and so there's this ethnic scholarship
306. and they said you can go for sure
307. you qualify
308. well she put down what her dad does for a living
309. and what his salary was
310. and they stopped talking to her immediately
311. so she'd been built up
312. saying that her ethnic background is going to let her do open all these doors
313. and let her go do this
314. and now she can't and she's pretty angry about it
315. so giving the wrong information can be can be
316. so I mean I'm worried that I'll never have all the information
317. Teacher: Yeah that's one thing that
318. you want to be careful with this kind of stuff
319. is that you want to make sure that you're giving accurate information
320. that you actually have good sources of information
321. and that what you're telling students is accurate

In this experience Jeff's niece had expected, based on what she had been told, that she would be able to qualify for a scholarship based on her ethnic background, but she did not receive the scholarship and was angry about it. However the key is in the final two lines of Jeff's comment. Both lines begin with "so" (ll. 315-316) indicating that Jeff is telling us the point of this experience. First he points out the problem of "giving the wrong information" and then amends it to say that he will "never have all the information" (l. 316).

Since a teacher can never have all of the information about an issue this lack of information positions him/teachers as helpless to act in these situations. The story also serves as an example of the futility of trying to help. I agree with his point about the need for accurate information, under the assumption that teachers still have the responsibility to find and share information. In contrast, Jeff seems to use the impossibility of gaining all of the information about an issue as a reason not to try, as disengagement from social

justice. This argument carries weight in part because we have consistently connected teacher knowledge to teacher authority and what it means to be a "good" mathematics teacher. In a situation when a teacher risks demonstrating a lack of knowledge and thereby undermining his authority then, according to this argument, it is better to avoid that possibility. In this way this story contributes to a discourse of ignorance as a reason not to engage in teaching mathematics for social justice.

As the discussion continues Jeff asks about what happens if incorrect information is given. In response I give an example from my own teaching and focus on the responsibility that teachers have to provide this information to their students if they have it.

322. Jeff: So we're talking about going to school
323. so I mean in Utah at least there's this law
324. so suppose they were told wrong or something
325. I mean the people running the program
326. don't always know as well as they might
327. Teacher: I in Colorado I had two students who were undocumented
328. um I taught them when I think they were juniors in high school
329. and when they were in middle school
330. somebody had told them that they would not be able to go to college
331. because they were undocumented
332. so they stopped trying
333. it wasn't true
334. and they didn't find out about it
335. until their junior year ok so
336. Gavin: So freshman and sophomore year
337. everything was working against them
338. Teacher: Their grades were much lower than they could have been
339. and so I think Lisa's point is a good one
340. you know what if no one else tells them
341. um and it's not I'm not saying it's your job to save the world kind of thing
342. but if you have information that could benefit students
343. I think you have a responsibility to share it
344. Gavin: Even if it is like
345. as Karl says like comes off as like
346. bringing them down as a person you know
347. they feel like they're capable and can do this themselves

348. Teacher: Sharing information is different than giving help I think
349. Gavin: No I agree with you
350. but like certain persons' definition of receiving information
351. being the same as getting help
352. differs from person to person
353. do you know what I mean

The example I give involves two of my former students who were undocumented. They had been told, incorrectly, that they would be unable to attend college. As a result their grades were much lower than they would have been otherwise and their chances to attend college were diminished. I use this example to emphasize that there is a chance that "no one else tells them" (l. 340) and that a teacher has "a responsibility to share" (l. 343); more specifically I am telling these teacher candidates that *they* have a responsibility to share information with their students.

Gavin clarifies my meaning to ask whether or not that includes the possibility of offending someone. When I try to distinguish between sharing information and being condescending he is quick to distance himself from the problem and place responsibility for the problem on the person offended. He quickly points out that he "agrees" (l. 349) with me about the differences between sharing information and being condescending. Gavin then refers to "certain persons' definition" (l. 350), which seems to be a coded reference to people of color, who may be offended regardless of his/the teacher's intent in sharing information. This positioning seems to suggest that, at times, there is nothing that the teacher/White person can do, because some people of color will choose to be offended (unreasonably). This may be connected to the dominant idea of "playing the race card" where White people accuse people of color of bringing racial offenses into a situation where (in the White person's judgment) there was no racist intent. Similar to my earlier linking of intention and responsibility, this discourse suggests that without racist

intent there is no racism. Again this becomes a reason not to provide information to students and diminishes the responsibility of teachers to their students.

Esperanza disagrees with Gavin's point and makes her case by relating some of her experience as a Latina, ESL student in order to once again reposition teachers as responsible to their students. In doing so, especially by bringing in her own experiences as a student, Esperanza counters the dominant discourse of students as passive and teachers as authoritative.

354. Esperanza: I feel like that's a very strong
355. Gavin: I guess I'm going
356. Esperanza: a risk you have to take
357. students as people
358. I think will take it either way
359. I feel like the conversations on immigration
360. um I am an immigrant and my family is an immigrant as well
361. like we're all immigrants in my family and stuff
362. and so I became very touchy when we talked about this in U.S. history and stuff
363. in high school
364. and I would get very upset about the debates that we had in class
365. and I there were moments when White male female teachers
366. were able to help me
367. and I saw a connection
368. oh they really want me
369. they care about me
370. but there were times when I took their comments as very like rude very upsetting
371. and I think as a teacher we have to kind of take the risk
372. I mean if we are doing it as a way to offer information to students
373. that might not know
374. if we're doing it as a way to acknowledge the discrimination or the injustice
375. that exists within our society
376. our best is to establish the discussion in certain ways
377. that our environment does not hinder with the student
378. that has gone through it in a spot that
379. does that make sense
380. we have to be very careful when we discuss any topic
381. because of just some the way you sound [((unintelligible))
Lines 382-401 ((Gavin recaps Esperanza's points, we also talk about teachers' relationships with students, and respect))

In response to Gavin's points, Esperanza quickly positions teachers as people who must

be willing to place the needs of students above their own. She points out there is "a risk you have to take" (l. 356). This comment is made generally about teachers, although the use of "you" (as opposed to "we", l. 356) makes it more personal to those of us in the room, and this may also direct her comment to Gavin specifically and White teachers generally. Further the "risk" she refers to is a personal risk on the part of the teacher. Within the context of this discussion it is a response to comments from Jeff and Gavin that students could take offense at White teachers' efforts to help them. By using "have to" (l. 356) she makes taking this risk an imperative, something that is necessary as part of being a teacher. This positions teachers as responsible to students, because they have information that students need.

Esperanza then gives an example from her life as an immigrant student. This example (as a kind of counter to Jeff's and Gavin's) demonstrates that teachers (including White teachers) can help and that students of color may "get very upset" (l. 364), but the teacher still has a responsibility to share that information. One of the keys for Esperanza was teachers who "care about me" (l. 369). As a result she reemphasizes her previous point stating, "we have to kind of take that risk" (l. 371). At this point she shifts her pronoun use to "we" and "our" (ll. 371-380). This seems to serve to make her statements more inclusive (not just addressing White teachers, but all teachers) and to make it more personal (this is for all of us in this room).

Esperanza then lists multiple ways that teachers can take the risk in order to be responsive to their students, in ways that will likely not be offensive. These include "offer information" (l. 372), "acknowledge the discrimination" (l. 374), "establish the discussion" (l. 376), and "not hinder with the student" (l. 377). Each of these verbs (offer,

acknowledge, and establish) suggests that the teacher is taking the first steps, but not being forceful. Thus the teacher is responsible to do these things, but still respectful of the students' feelings and perspective. These suggestions position students and teachers in a balanced relationship. Finally she urges teachers to "not hinder," in other words to allow the student to participate and make decisions. She closes out this section of her comments encouraging teachers to "be very careful" (l. 380) because of "the way you sound" (l. 381). This pronoun switch back to "you" may serve to point out to White teachers in particular that they must be careful in how they talk about issues that are connected to race. Unfortunately Gavin began talking over her and we do not know what she was about to say.

Jeff now shifts the discussion by asking Esperanza more about her own experiences as a student. In doing so he uses White ignorance (a form of helplessness) of racial issues to suggest that he/White teachers would be better off not having difficult discussions to begin with. This is a continuation of his previous suggestion that because of the impossibility of knowing everything, it may be better not to try.

402. Jeff: So just a question for Esperanza on
403. so if a teacher
404. because one of my fears is
405. I don't see I don't see when I'm doing it wrong for them
406. so but if I said look I want to have a discussion about immigrant reform or
407. whatever it is the topic was
408. and I know that I'm not an immigrant
409. so I'm not going to see all of the issues
410. the way that you will
411. and so if I if I do something you know prelude
412. that says I'm going to probably screw some of this up
413. but it's not because I'm trying to
414. would that have made a difference for you
415. Esperanza: Letting me know yea
416. and I think it would be great to not take on and shut
417. because I know my U.S. history teacher he thought he knew everything

418. he just knew the newspaper
419. like you're not looking at the families that are being deported
420. you're just bringing the issues the media brings out to the table
421. and not other perspectives
422. and so if you really want to do touchy subjects like that
423. I feel like you have to make sure you know both sides of the coin
424. you know what I mean like
425. Teacher: Yeah and I think like you said
426. acknowledge the limits of your knowledge
427. Esperanza: Exactly let them know
428. Jeff: I don't know
429. at some point then I might just decide not to have it
430. because the problem is is
431. that if there is something that the teacher brings to the class
432. and if nothing else it's some sophistication in how debates can go
433. and realizing that kids just like to argue
434. and so what are they going to do
435. they're going to bring the news to class
436. and their parents views
437. and so if you aren't knowledgeable you can't manage that discussion very well right
438. I mean if I personally am not knowledgeable about it
439. it can get out of hand very quickly.

As Jeff directs his question to Esperanza he frames it around his own fear (l. 404) which is, "I don't see when I'm doing it wrong" (l. 405). It is important for teachers, especially White teachers, to recognize the limits of their knowledge and experience. However, this recognition should be accompanied by a sense of responsibility to learn more and prepare to have difficult discussions, rather than an excuse to not have those discussions at all. It is not yet clear which direction Jeff will take this discussion, however, this question begins the discourse of White ignorance of racial issues and will continue through this section of the discussion.

Further as Jeff has framed the problem here he is drawing on a clear distinction between types of knowledge. If a mathematics teacher lacked some aspect of mathematical knowledge necessary for teaching, he or she would be expected to learn it; that is part of being a good mathematics teacher. However, when the lack of knowledge

is about students and their experiences, especially students of color, the same expectation does not hold. In this way priority is given to mathematical knowledge rather than knowledge of students. This hierarchy of knowledge grows from the dominant discourses of mathematics education.

In Esperanza's response she emphasizes that a teacher should "know both sides" (l. 423) of an issue. This would require a teacher to learn about issues before discussing them in class and positions the teacher as taking responsibility. It is at this point that the direction of Jeff's argument becomes clear. In response he says, "I might just decide not to have it [the discussion]" (l. 429). Thus his ignorance has become a reason not to take on difficult topics rather than a responsibility to learn. He links this to the idea of "sophistication" (l. 432) that a teacher should bring to the class. This sophistication requires more knowledge than what the students have (who "just like to argue" l. 433) in order to give the teacher the authority to control "how debates can go" (l. 432), since "if you aren't knowledgeable you can't manage that discussion" (l. 437). The concern is that the discussion "can get out of hand" (l. 439), which undermines teacher authority. In this way White ignorance leads to a fear of appearing to lack authority. This fear prevents a willingness to learn what is required in order to have the difficult discussions that could benefit students. It is a fear of not being a "good" teacher.

Throughout our week 12 class the positions for teachers and students shifted back and forth. When the teacher candidates successfully positioned students as intelligent and capable the role for teachers became one of responsibility to share information with students and respect their needs and feelings. However, whether in response to fear, desires to be "good" teachers, or an inability to escape the link between knowledge and

authority, our discussions continued to push teacher subject positions back into the traditional roles. This was accomplished by positioning the teacher as authoritative by reducing the teacher's role to mathematics or to areas where the teacher is more knowledgeable than the students. At other times this was accomplished by positioning the teacher as helpless because of the potential anger of students of color or because of White ignorance about racial issues.

Balancing Teacher and Student Authority

As Karl reported on his action research project in our week 13 class, he shared with us an example of when he had brought up a social justice issue in his class. The opening section of this report was analyzed in Chapter 5. Here I reanalyze this same section and what followed focusing on subject positions. Karl's action research project focused on teacher authority in the classroom. He noticed in his teaching that when he said what he thought about a mathematics problem or idea that his students would stop offering their own ideas. In an attempt to change this and have students develop more confidence in their own thinking, he began changing the way he taught his classes to give more time to students to talk about their work with each other and with their class. The selection that follows is from his second report on his project. In this report he describes how his teaching has changed and how his students' participation has changed. He positions his students as active participants in the class and himself as still authoritative, but, rather than use authority to maintain his own authority, he uses authority to encourage his students.

1. Karl: Ok ok report number 2
2. um so uh in as a recap
3. my topic is kind of in general authority in the classroom

4. I mix up which ones people are doing
5. so just to refresh
Lines 6-63 ((Karl explains some of the difficulty he has had getting feedback and describes how he modified an assignment for his class))
64. we calculated the area based on those grids that you put over the maps
65. and then talked let uh used that as a segue into
66. you found the area it looks like
67. you guys used a rectangle here this one
68. you guys said was a half what's really going on
69. so like that was kind of the math reason for doing it
70. but the discussion was pretty good
71. it started out kind of poorly I thought
72. just because maybe I didn't I didn't expect the response initially to be like well
73. to start talking about this
74. and students would be like basically say I don't give a s____ about parks
75. like why are we doing this
76. I don't care like to be kind of like
77. well what would be fair
78. and they would say let's just have it how it is now
79. I don't care I don't care what parks are like
80. and that was about the first five minutes of the discussion was about
81. how people don't really care about parks

Here Karl is explaining about a lesson he did that was originally developed by Esperanza, Lisa, Jane, and Stella that illustrates the ways parks are distributed across our city in inequitable ways. In this section Karl's focus is on what the students did. Notice how he includes himself with his students ("we calculated" l. 64) and then the frequent use of "you" (ll. 66-68) to refer to his students and the work that they were doing. In this way he positions students and their work as central to the class, while he remains on the periphery. Then as he moves into how the lesson "started out kind of poorly" (l. 71) he attributes this to his own inexperience (l. 72) rather than to some student attribute. He emphasizes the students' perspective and identifies with it by putting it in first person (ll. 74, 76, 79). While their perspective went against what he was trying to do he listened to and valued their thinking.

After listening to his students, Karl still thought that the lesson was valuable and

that his students would benefit from it. As a result he attempted to persuade them to give the lesson a chance.

82. and I had like a little bit of a discussion a little bit of like a talk
83. like a little bit more serious like
84. hey like this I know we're talking about parks right now
85. but take this a little more seriously for a little while
86. this isn't really about parks
87. and I think you guys will figure out what's going on here
88. and we talked about that a bit and the conversation got really good
89. and some things that I mentioned to
90. I think the two of you yesterday
91. that they brought up were things
92. that I never thought about in terms of this park uh issue
93. and one thing being cost
94. or a couple things being cost of upkeep for a park
95. I didn't really think of that for as a possible reason
96. for maybe how how they're distributed
97. like maybe one park has more graffiti than another park
98. and more cost is gone into upkeeping it
99. one student brought up the distinction between a community park and a city park
100. and I didn't know the difference really
101. and she was kind of thinking she was like schools are kind of supported
102. and I don't know much about taxes
103. and she was kind of asking me about taxes
104. and I was like I don't know your idea could be right
105. or it could be wrong
106. so she was kind of wondering if taxes from a certain community
107. kind of go into parks
108. or if it's all everything from the city goes into one big fund
109. and then it gets distributed into all of the parks
110. or for instance there are some parks in Salt Lake that are funded privately
111. or from say the Sugarhouse community
112. which I thought was an interesting question
113. that I didn't expect from a 17 year old
114. but I think in terms of authority I think it was a really good thing to talk about that
115. and we did even bring up that from there
116. I kind of talked about that maybe you mentioned someone mentioned that the
117. um the way to get in state tuition in Utah as opposed to like other states
118. and so we brought that up
119. and we had a short discussion on that and
120. students were really interested in that
121. see this is kind of like more what this parks discussion is about
122. like this is an opportunity in Utah
123. this is these other states keep thinking about this

124. so just wanted to talk about that as an aside

In the beginning part of this section Karl portrays himself as using some of his authority to persuade his students to continue with the lesson. However, he did so through "discussion" (l. 82), recognized the students' perspective (l. 84), and expressed confidence in their ability to see beyond the parks (l. 87). The result was that students brought up things, "that [he] never thought about" (l. 92). Karl clearly views this as a positive (it immediately follows his evaluation of the conversation as "really good" l. 88) even though it positions him as less knowledgeable in this area than some of his students. He believed that his students' ideas were valuable enough to share them with the rest of us.

Karl also openly admitted his own lack of knowledge on the subject to his students (l. 104). In previous discussions we had constructed teacher authority as closely connected to knowing more than students, this admission appears to be an attempt to work against that link between knowledge and authority. However, this admission was not related to mathematical knowledge and a mathematics teacher's authority is most closely tied to her or his mathematical knowledge. It is possible that Karl feels safe admitting a lack of knowledge about park funding, but may not yet in a mathematical realm. In either case, it seems that introducing a mathematics lesson that addresses equity shifted, at least momentarily, the power relations of the class.

At this point Karl changes the focus of his discussion to talk about the other class he was working with. In previous reports, Karl has explained that this class (Secondary II) is significantly different from the one he just discussed (Geometry, which was being phased out). Secondary II was much larger, had a higher proportion of White students,

the students were less amenable to presenting, and he felt more restricted in terms of the curriculum. Possibly as a result of these circumstances he focuses much more on what he is doing and less on the students, their thinking, or their actions. In this way he returns to the dominant discourses that center teacher action.

125. in terms of secondary II
126. I've been trying to talk more about this whole authority thing
127. um and in that vein I've been having students present more often
128. I'm kind of like easing them into it
129. they've been doing just little presentations with groups
130. like going up in a group of three group of four in front of the class
131. and talking about things
132. in general people have been supportive I think
133. and I haven't been
134. courtesy of Gavin and Jeff's discussion at the conference
135. which I thought was really really helpful
136. I haven't been as pushy with with some students in terms of
137. uh as like a cultural thing
138. like some students may not
139. because of their culture want to present in front
140. or feel like a teacher
141. or feel like they are the most knowledgeable teacher about something
142. so if a student is like
143. I don't want to talk in front of the class
144. I just that's fine just come up and stand with the group
145. and if they want support you're there to help them
146. and it’s gone pretty well

In the first three lines Karl positions himself as the agent (“I’ve been trying” l. 126, “I’ve been having” l. 127, “I’m . . . easing” l. 128). The students are positioned as more passive. After mentioning briefly how his students presented in groups he again focuses on his efforts to not be “pushy” (l. 136). This can be understood as holding back authority in order to be sensitive to cultural differences. This shows a more respectful relationship between teacher and students, but only focusing on the teacher’s side of it. Here the emphasis seems to be more on how Karl uses authority in an attempt to change the relationship between teacher and students. At other times he has explained that his efforts

with Secondary II have been less successful, so without success stories to relate, the focus shifts to what he is trying to do.

Conclusion

During the course of the classes reviewed here the subject positions we described for ourselves and our students changed and shifted in multiple ways. Often, as we invoked some of the dominant discourses of mathematics education and teacher authority these discourses pushed us into traditional ideas of what it means to be a teacher or a student, and how authority is used in these relationships. The teacher candidates positioned themselves as having justifiable authority over their students as a result of their greater knowledge and experience. This authority allowed the teacher candidates to make decisions about their teaching and their students without fully considering the needs and desires of their students.

When our discussions included social justice and equity the subject positions for teachers shifted. In these cases we either left teachers out of the discussion (implying that the discussion did not apply to teachers) or constructed subject positions for the teacher that were helpless (lacking authority) to apply what we were discussing to our teaching. This helplessness served to excuse teachers from the responsibility to consider the social justice needs of their students and to deflect the blame that accompanies discourses of individual responsibility. The desire to deflect blame reflects a situation where we felt we could not be playful; in these situations we took ourselves and our image as "good" too seriously to risk being a fool. This occurred in part because we had not developed a meaningful position for students in the classroom or an understanding of authority as shared. Helplessness was constructed by presenting teachers as lacking knowledge,

lacking experience, ignorant of racial issues, or controlled by others (either administrators or student anger).

Finally as we considered complete subject positions for students as intelligent, capable, and active, the teacher candidates and I created teacher subject positions that reflected a feeling of responsibility to our students. These subject positions required teachers to have authority, but to be aware of and take responsibility for the effects of that authority. Further we should use that authority to improve the educational opportunities and outcomes of our students. This subject position could be further developed if we recognized the various needs that students have including racial, cultural, gender, and linguistic differences.

CHAPTER 7

LIVING IN THE STRUGGLE

Based on prior work on mathematics and social justice (Bartell, 2013; de Freitas, 2008; Gutstein, 2006) I expected some if not all of my students to be resistant to efforts to combine mathematics with social justice education. I was surprised to find little overt resistance to these efforts and at multiple points every one of my students expressed a desire to incorporate principles of social justice into their teaching. Despite the clear disagreements brought up primarily by Jeff and Gavin and which were presented in previous chapters, in the moment and flow of the discussion I interpreted these as sincere concerns, questions, and misunderstandings with a desire to understand. All of the teacher candidates demonstrated an ability to critique society, mathematics education and their own practice in ways that are consistent with social justice aims. The challenges we had in understanding what it means to teach for social justice with mathematics illustrates the power of normative goodness. By following the dominant discursive "rules" we can present resistance without seeming to, at times even to ourselves.

Not surprisingly one of the main areas of struggle for all of us was overcoming the abstract nature of school mathematics that we experienced as students and that we felt as an expectation from administrators, parents, students, and ourselves. For most of us social justice was easiest to talk about in the abstract, but this also made it harder to apply to specific teaching contexts. For some teacher candidates the abstraction of mathematics

was considered a key characteristic that had its own value in facilitating communication of ideas. We had several discussions in which this came up and it became clear that many of the teacher candidates valued and defended abstraction as something that all students needed to learn. They often placed this above other crucial considerations such as equity or student understanding. While there were plenty of times when the mathematics itself was the source of struggle, there were other times when just the basic expectations of being a teacher were the cause of struggle and contradiction. While there certainly are cases when teaching itself does not fit well with social justice, I argue that in many instances the pressure is specific to mathematics teaching and that a teacher in another field would not have felt the same contradictions between a desire to teach for social justice and the expectations of their profession.

The teacher candidates struggled to reconcile their desires to teach for social justice with their conception of what a mathematics teacher should be. They felt constrained by the expectations of administration, peers, and curriculum to maintain specific subject positions. The need to meet these expectations pressured them to position themselves as relatively powerless to make significant changes to the ways in which mathematics has been taught for decades and the ways in which mathematics classes are used as a means of excluding women and students of color. At other times the teacher candidates positioned themselves as powerful and authoritative and their students as relatively powerless. In these cases then the teachers viewed themselves as simultaneously powerful and powerless, rather than potentially both powerful and powerless and students as similarly powerful and powerless. They saw themselves as powerful in the sense of their knowledge, abilities, and their teacher role in the classroom

and powerless to make changes towards social justice because of the constraints of discourse. Even Karl who explicitly addressed teacher and student authority in his work viewed himself as successful in only one his classes with a less rigidly defined curriculum and that was not attached to any end-of-year test, thus reducing the pressure he felt to conform to those discourses.

By focusing this chapter on struggle, as opposed to highlighting success or progress, I am drawing on particular understandings of Whiteness that recognize the difficulty of social justice work and the potential for social justice work to also reinscribe Whiteness. As Thompson (2003) explains, White, antiracist academics can imply that there are authoritative, "right" ways to be antiracist or to do antiracist work. This occurs as we critique the efforts of our students to engage in discussions of antiracism. We point out that they do not "get it," and, in passing judgment on their efforts we suggest that, in contrast, we do "get it." The problem is that in doing so we maintain a hierarchy of what it means to be a "good" person, placing ourselves farther along the continuum than our students and suggesting that there is some "right" way to do social justice work. Gutiérrez (2015) suggests that as teacher educators we can unintentionally push our students to "get it" when we try to rush them to resolutions and conclusions rather than allowing them and supporting them in holding on to "nest in" the uncertainties and tensions of teaching for social justice.

As explored earlier Applebaum's (2010) conception of vigilance provides one potential way of working against dominant discourses. This vigilance includes *uncertainty*, *humility*, and Foucauldian *critique*. *Uncertainty* includes a willingness to question our own certainty and what we think we know, especially about racism and the

experiences of others. Uncertainty is a counter to the certainty of White judgment, especially of what does or does not count as racism. However, it is also a counter to the certainty of abstract mathematics. Thus a mathematics teacher purposely creating a moment of uncertainty by teaching a social justice lesson (as when Karl did) can require an openness to surprise that is stronger than the fear of the uncertain. *Humility* means that we are "open to examining how our progressiveness might be oppressive in ways that we are not aware of" (p. 186). This kind of humility requires critical self-examination. Lisa invited us to do this kind of work when she suggested that perhaps our students of color are right not to trust their White teachers. *Critique* includes critique of power systems, but importantly extends to critique our own frameworks of knowing; to ask ourselves what our assumptions make impossible for us to know or question. As we discussed mathematics and social justice education we regularly found ourselves stuck in binaries of pure and applied mathematics. These binaries, in a sense, make it impossible for us to understand social justice mathematics that addresses identity or power, since these are neither pure nor applied.

The purpose of this chapter is to show how we each struggled in different ways with some of the dominant discourses of mathematics education and Whiteness. The point here is not to show that these teacher candidates now understand social justice mathematics or that they have somehow resolved the challenges of this work. Instead, the point is to show that (beginning) mathematics teachers can engage in these efforts. In this chapter I focus principally on the teacher candidates' written work, which included a weekly journal of their reflections about their student teaching experience and their final report of the action research projects. Often the teacher candidates were more willing to

be open about the challenges they were facing if they knew that their peers were not reading what they wrote. This may be similar to my own reticence to bring up what I consider to be the more radical aspects of my research in mainstream settings. In this way this openness may be less about their peers than it is about the formal and institutionally sanctioned classroom space. I have examined each of these documents and selected those portions where the teacher candidates expressed some kind of struggle between issues of social justice and mathematics. In what follows I present excerpts that show how I struggle with Whiteness to talk about the more radical aspects of my research, how Karl, Stella, and Jane each struggle, in different ways, with their authority as teachers, how Lisa struggles to balance an achievement focus with student identity, and how Esperanza struggles with trying to be accepted as a "good" mathematics teacher and maintain her Latina identity. Esperanza's clear writing about the effect of race on her efforts to become a mathematics teacher highlights (by contrast) the way Karl, Stella, Jane, and I do not meaningfully grapple with our own Whiteness in our teaching. While we may understand that our Whiteness shapes us and our teaching, we only address this superficially, if at all.

As in previous chapters I draw on Gee's (2005) building tasks. In particular I use the building tasks of "Significance" (what the speaker means and makes relevant), "Activities" (what the speaker intends to do through discourse), "Identities" (how the speaker positions him- or herself and others,) and "Relationships" (how the speaker constructs relationships between her- or himself and others and between others). Previous chapters used others of the building tasks, but these four are the most relevant to show how the teacher candidates and I struggled to understand teaching mathematics for social

justice.

Radical Research in Mathematics Education

Trevor—Teacher

Silence

The discourses of mathematics and mathematics education are constructed to exclude the possibility of bringing in social justice perspectives. Consider the section below from my reflection journal[20] that I wrote after attending a presentation at the newly formed National Council of Teachers of Mathematics (NCTM) Research Conference 2014 (formerly the Research Presession). I had just finished listening to a discussion between a few scholars (Danny Martin, David Stinson, and Maxine McKinney de Royston) who work in mathematics education and social justice. The discussion was fascinating and connected with my work in important ways. I took a few minutes afterward to talk to Danny Martin and then wrote what follows.

4/8/2014
Lines 1-5 ((I explain where I am and the session that I've just listened to))
6. First I realized yesterday that as I've practiced explaining my dissertation topic to
7. people outside of math ed and critical perspectives that I leave out the part about
8. Whiteness because it would be too difficult/long/politically challenging to explain. In
9. these kinds of places and with Dr. Martin in particular I shouldn't do that and there is
10. no need to do that.

Important in interpreting these comments is understanding the context that I was writing in. The National Council of Teachers of Mathematics is the largest and most influential professional organization of mathematics educators in the United States (Gutiérrez, 2015). They exercise significant influence over national mathematics standards (such as

[20] During the semester when I taught the class that is the focus of this study I kept a reflection journal of my thoughts about my teaching and related ideas.

the Common Core, Wolfmeyer, 2014) and have published their own set of mathematics education principles, the first of which addresses equity. These principles influence mathematics teachers as well as mathematics teacher educators and researchers. The NCTM publishes the highest ranked U.S. mathematics education research journal. However, the NCTM approach to equity has been criticized among social justice mathematics educators (Gutstein, 2003; 2009). Among the nearly 200 presentations at this conference only three explicitly addressed social justice (two of which were posters) and one addressed racism (the session I attended above) in mathematics education.[21]

Clearly social justice and racism in mathematics education research were largely absent from this largest organization of mathematics educators. Thus as other conference attendees asked about my own research I often left out the more critical, poststructural aspects (such as Whiteness Theory), and the accompanying more nuanced understandings of social justice out of my description of my research. This occurred to the point that even when talking with Danny Martin, who would have understood and appreciated these aspects, within the context of this conference I "leave out the part about Whiteness" (ll. 7-8). I was reminded of this nearly a year later as I sat in on an interview for a mathematics education position at the university I currently attend. When it came to my time to introduce myself and what I do I just mentioned mathematics education and social justice even though the scholar we were interviewing also used poststructural methodology and Whiteness Theory, in his research on mathematics education. It was not

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[21] There were a few other sessions which also addressed social justice, but they did not proclaim it in their title or abstract, perhaps a conscious choice to increase the likelihood of acceptance. I have not included sessions that addressed equity, since it was impossible to tell from the program description if they were using NCTM's version of equity or something more in line with Gutiérrez (2012c).

until the interviewee mentioned the difficulty he has in bringing up those aspects of his research with mathematics education circles that I realized that I had just done the same.

The point here is not my timidity in bringing up the more complicated and radical aspects of my research, but rather to illustrate that the discourses of mathematics education silence these perspectives, because of their absence in national discussions and positioning on the radical fringe of mathematics education. This leads to situations where in our first class as the teacher candidates asked me about my research I left out large parts of what my research is really about.

6. Teacher: No its fine if you know,
7. in fact it might be better.
8. Uhm I'm just trying to figure out how to do it in a short
9. rather than since I just wrote 60 pages about this.
10. How to do it briefly instead of uh larger.
11. Uhm the basic idea is that
12. the way we think of mathematics
13. now in modern times
14. is often very abstract formal uh symbolic
15. all that kinda stuff uhm generally apolitical, acultural
16. uhm those kinds of things
17. and then you have this group of people
18. who are practicing social justice math
19. which is political, contextualized
20. Uhm so it seems like
21. these two things can't really mix very well.
22. The idea of mathematics and social justice.
23. Uhm but historically that's not how mathematics has always been.
24. It hasn't always been abstract
25. and so formal as it is now.
26. uhm at least how it is now taught in schools.
27. And so what I am interested in looking at
28. is how people talk about mathematics and social justice
29. in ways that make them not fit together very well
30. and then how kind of we can recognize
31. what's happening when we talk about mathematics and social justice
32. and try to merge the two
33. in a way that makes sense and is workable.

The discussion continues along these lines for a few minutes, but here and throughout I

focus almost entirely on the idea of abstract mathematics and that there could be other ways to think about mathematics. I briefly mention social justice, but make no mention of Whiteness, racism, or poststructuralism. Later in the semester we did talk about Whiteness, but only rarely, and racism more frequently. Notice also in this section how frequently I hesitate (I say "uhm" a total of seven times in this section) and how frequently I qualify my answers. This is unusual for me in teaching contexts and indicates that I am being careful in how I phrase things and in the words that I am choosing, much more so than in most of the rest of the class.

I knew that I was working within a dominant mathematical context with teacher candidates who were well versed in dominant mathematics. I knew that within this discourse there was no place for explicitly political concerns or for the uncertainty and nuance of Whiteness Theory and poststructuralism. I was concerned that the teacher candidates would tune me out before the course really got started and as a result I silenced those portions of my research. I was also concerned that if I voiced my positions too strongly that I would silence their views. In these ways I was trying to start off the class by positioning myself as a "good" teacher. In doing so I tried to conform to what I thought these teacher candidates would expect from a professor. As a result I tried to hide the more "radical" aspects of my research.

In my initial analysis of this section I conveniently made no mention of my own Whiteness and the discomfort that I sometimes feel in addressing the racial aspects of my identity in talking to colleagues. This lack of attention to my own Whiteness points to the power of these discourses at normalizing the idea that race only belongs to people of color and that Whites (as a result) do not have race. Thus it is not just that these

discourses and the contexts that I was working in exclude social justice and radical perspectives on mathematics education, but also that within these spaces and these discourses a White person talking about themselves as raced is an uncomfortable disruption. As a result part of my reticence in talking about Whiteness specifically was to avoid the personal discomfort of disrupting these discourses and to avoid causing discomfort to my White students (especially in the first class).

Teachers, Students, and Authority

Karl

Questioning Authority

Early in the semester, before we talked about authority in class Karl began questioning the ways that his authority affected his teaching. In his student-teaching Karl worked with a very capable and mostly traditional mentor teacher. While his mentor was traditional he encouraged Karl to try out different ideas, including those that address social justice. The two (Karl and his mentor) worked very well together and team-taught effectively. Karl taught two classes in a large, diverse public high school. One class was smaller and primarily composed of students of color, many of whom were ELL students and/or refugees. The second, larger class included more White students, but was still diverse. I believe this supportive context gave Karl confidence to try things out and to question his teaching practice in ways that many of the other teacher candidates did not feel they could. Additionally his work with students of color may have heightened his awareness of his use of authority. This seems to lead Karl to question his use of authority. In this early journal entry as Karl expresses his discomfort with his authority he also places the responsibility for this on his students.

1/27/14
1. Another thing that I experience as I am teaching is a strange sense of authority. Since I
2. am the teacher the students seem to assume that my opinions and beliefs about
3. mathematics hold more weight than their own.
4. To me this seems very wrong. I am not saying that students should necessarily be
5. confident in their original mathematical ideas, but I need to find a way to create an
6. environment where students construct their own beliefs and opinions about
7. mathematics. Another thing that really bothers me is the formality that mathematics
8. classrooms can have. Though it is necessary to have formal definitions and rigor in
9. mathematics, there also needs to be personality. Students should be able to embed their
10. own slang, language and definitions into mathematics. This has been discussed in
11. many of my classes and I urge students to construct definitions for themselves.
12. Students though seem very wary at the idea.
13. I will continue to encourage students to use slang, and their native language in talking
14. about and working with mathematics. Creating an atmosphere of valued ideas will
15. also help students build confidence in expressing original thoughts.

In this short section Karl brings up two main problems that he sees, first is the authority (l. 1) that he feels as a teacher and second is the formality of the mathematics classroom (ll. 7-8). While he does not explicitly link authority with formality their proximity in his writing suggests that they are possibly connected. He explains that authority is a problem because the students give "more weight" (l. 3) to his ideas than to "their own." Karl is clearly uncomfortable with the authority that he feels he has as a teacher ("strange sense of authority" l. 1; "this seems very wrong" l. 4), but he places the responsibility for this authority on his students who "seem to assume" (l. 2) his authority. To change he wants to create a classroom "where students construct their own beliefs and opinions about mathematics" (l. 6). This desire connects with the identity and power aspects of equity as outlined by Gutiérrez (2012c). However, this change requires that the students participate in their education and that their voices carry greater weight in the classroom, two positions that do not fit with the dominant discourses of teacher authority, especially as it connects to mathematical knowledge. What Karl leaves unsaid here (although he brings it up later) is how his authority is also connected to being a White male. Thus the silencing

effect of his statements on his students may be greater than it would be for other teachers.

To achieve what Karl suggests here students would need to make mathematics their own and to be confident in their own ideas and thinking. This desire seems to be contradicted by "the formality that mathematics classrooms can have" (ll. 7-8), which is the second problem. The passive construction seems to suggest that Karl sees this formality as inevitable and out of his control. This sense is reinforced as in the next sentence the necessity of "formal definitions and rigor" (l. 8) is placed at the front, marking it given information (Karl assumes the reader will not disagree with this), while the necessity of personality is placed at the end, suggesting that agreement about personality cannot necessarily be assumed. It seems then that Karl is struggling to incorporate a couple of novel ideas (student thinking and personality) into the structure of how he understands mathematics classes (where formality and teacher authority are assumed). While he expresses a desire to bring these into his teaching he continues to position himself as agent within the classroom (ll. 11-14) and as a consequence to limit potential student participation. Throughout this section Karl is both questioning and recentering his authority as teacher. He is uncomfortable with this authority, but does not yet have a way to understand students as also powerful/authoritative.

In a later journal entry Karl explains his decision to address his authority through his action research project and how this project connects to equity. As he does, Karl explains how he sees the value of student contributions to his class.

02/24/14
1. I have chosen to explore teacher power and authority through action research. As
2. teachers we are a very powerful force in the classroom, and we are seen as the ones
3. who should be speaking, respected, and who have the most important ideas. While we
4. are expected to know the content, it is the students who present the ideas that are most
5. valuable to the classroom. Ideas that are incomplete, partially correct, entirely incorrect

6. and incomprehensible are very valuable for many reasons. Consider if students only
7. ever saw mathematics as the coherent presentation of ideas. How is a student supposed
8. to feel empowered in the classroom if this is how math is known? A student should not
9. be expected to discuss mathematics in this way, so we should not have the classroom
10. structured in a way where mathematical presentations are always flawless.
Lines 11-20 ((Karl mentions some ways in which ELL students benefit))
21. Another way in which power addresses equity is that it takes me a little bit out of the
22. spotlight. By giving students more power we can focus more on who each student is
23. as an individual, and focus less on who I am (a white, middle-class, college-educated,
24. male teacher.) Students need role models in the class that they can connect with to
25. show that they can be mathematicians as well. What I am discussing now is bringing
26. culture into the classroom – but I do believe that this is related. By showing students
27. that there are many mathematicians of different races, different genders, and different
28. socioeconomic backgrounds this distributes teacher power to more individuals. As a
29. teacher I will try to emphasize that there are many ways of thinking about
30. mathematics, and that they are all important.

This time as Karl addresses authority he recognizes the power that teachers have (l. 2) and does not place the responsibility for this on students. Instead he uses the more open phrase "we are seen" (l. 2) to explain where this power comes from. This passive construction probably includes students as some of those who see teachers in this way, but it also can include other teachers, administrators, and parents among others. Karl then directly challenges this view of teachers, suggesting that students have the most valuable ideas in the classroom (ll. 4-5). Part of this value comes from the implication that the ideas from students will be "incomplete" (l. 5), etc., because this will give students a more realistic expectation for their mathematical ability and so that they "feel empowered" (l. 8). Karl views these as a counter to the "flawless" (l. 10) presentations in traditional mathematics settings. By questioning his authority Karl has envisioned a larger role for the students in his classroom.

While Karl never says what having students present their ideas is valuable for, we can assume that he believes that this will help students to better learn and connect with school mathematics. The contradiction here is that students' mathematical ideas are

portrayed as valuable not for their quality or creativity, but for their imperfection. In this sense then they have pedagogical, but not mathematical value. This lack of mathematical value positions students as incapable of contributing mathematically relevant ideas. Thus Karl critiques the ways in which mathematics education often excludes students through the emphasis on correct answers (l. 7, l. 10). However, his framing of student contributions also excludes their mathematical value. The importance of making the classroom a safe place for students to make mistakes is not at issue here, however, we must also allow for the possibility that students' ideas may be mathematically valuable, even in their incompleteness. Dominant discourses of mathematics education do not allow for this possibility.

As Karl begins to write more specifically about equity he suggests that by focusing less on himself/teacher he can focus more on the students (ll. 21-24). This shift in focus comes "by giving students more power" (l. 22). By using the verb "giving" Karl suggests that he holds the power and that he chooses to give this power to students. As such he could also take it back. In this way he retains some level of control over his students. He then points out his own privileges suggesting that, because of them, he may not be a role model that students can connect with (l. 24). As a counter to his Whiteness he suggests "bringing culture into the classroom" (ll. 25-26). Thus Karl recognizes the problems created by his Whiteness, including his authority as a teacher. However, he is unable/unsure of how to work with this power and his students to create a powerful subject position with his students.

Stella

Learning to Center Students

Similar to Karl, Stella describes a change in her thinking regarding how she works with her students. Stella did her work in a small, diverse mathematics and science focused charter school. Her mentor teacher regularly used progressive reform teaching methods, but did not necessarily extend into social justice. She was supportive and encouraging of Stella, but also expected her to closely follow the textbook for the class. This mixture of support and limits helped Stella develop her ability to use reform methods, but may have limited what she could do in terms of social justice. Her writings seem to reflect this support and limits as they focus mainly on her learning about reform values and beliefs. In this journal entry Stella describes how she learns from her students to better meet their needs and not overstep her own use of authority.

WEEK 5 2/18/2014
Student Teaching Reflections:
1. I had a great time with student teaching this week. I've realized lately that when I'm
2. struggling to manage my classroom, I get sort of tense and micromanage my students
3. too much.
Lines 3-10 ((Stella explains balancing student participation with keeping the lesson moving)).
11. I've also started to realize that off-task behavior is not always what I think it looks
12. like. For instance, some of my students listen better when drawing, and for some
13. writing notes down seems to be more of a hindrance than a help, etc. While I think it
14. is important to do some note writing, I shouldn't require all of my students to write
15. everything down, or to be sitting up perfectly and watching me while I talk, etc. That
16. would be insisting that students learn equally rather than equitably. I've realized that I
17. came into teaching with a picture of what learning looks like, and in some cases I've
18. tried to force that on my students. However, in reality, the best learning environment
19. for a student might look different than the picture in my head. I've tried to listen more
20. to my students, and observe the bigger picture more than the small behaviors. For
21. instance, one day this week I experimented with a student and tried to give her more
22. freedom than usual. I usually ask her several times throughout the class to get her
23. notebook out and write things down, even though her homework and tests show that
24. she is keeping up well with the material. I let her go throughout the class without
25. asking her to write things down, and I noticed she was more attentive and actually

26. wrote a few key things down in her notebook. Perhaps this is a better way of learning
27. for this student. Listening to students in this way is less stressful for me (because I'm
28. not trying to manage their every action) and I think more beneficial to their learning.
29. While allowing this kind of freedom may not be possible in every case (some
30. behaviors may be detrimental to a particular students, or even to the class as a whole),
31. I think getting to know your students in this way is an important part of teaching!

Stella starts out by describing her tendency to "micromanage" (l. 2) her students. She links this tendency to her ability to manage her classroom (l. 2). In linking these two she takes responsibility for this struggle (drawing on dominant discourses of responsibility) in her teaching rather than laying blame on her students' behavior requiring "micromanagement." This led her to question what "off-task behavior" (l. 11) is. As a result, instead of trying to be more controlling of student behavior, she tried to exercise less control. She describes her previous ways of thinking as using her authority to gain compliance ("require all of my students to write everything down" ll. 15-16 and "I've tried to force that on my students" ll. 17-18). In contrast, now Stella "tried to listen more" (l. 19), "tried to give her more freedom" (ll. 21-22), and "let her go" (l. 24). However, even in these new ways of thinking, which are more respectful to students, Stella positions herself as the main authority since she is the one giving freedom and allowing (or not) different student behaviors. This freedom for her students is conditional and will continue when the results ("more attentive" l. 25 and "wrote a few key things down" l. 26) meet Stella's approval. In this way Stella is the judge of what are acceptable ways of participating in her class, but she also is questioning the way her practices, in particular her use of authority, affect her students individually.

As Stella continued her student teaching, her thinking again shifted in regard to her use of authority in managing student behavior. This time the shift occurred as she learned from watching other teachers at her school.

WEEK 7 3/3/2014
Lines 1-6 ((Stella describes how parent meetings are set up at her school))
6. I was able to offer
7. suggestions for how we could help Amy succeed in my classroom at the beginning of
8. the meeting, and as the meeting went on I was struck by how much the other teachers
9. were asking Amy questions rather than trying to offer suggestions for help. Most of the
10. meeting was spent asking her how she was feeling about her classes, why she was
11. engaging in different behaviors, if she had friends and who they were, if she was
12. happy at this school, how they could help her in their classes, etc. I thought it was
13. so cool that it seemed like these teachers called this meeting in order to better
14. understand Amy so that they could meet her individual needs, rather than going into
15. the meeting assuming they understood Amy based on her observed behaviors (as I
16. did!).
17. I definitely learned a lot from observing these teachers interact with Amy and her
18. parents. I feel like I say over and over again that I want to get to know students and
19. understand their individual needs (i.e. teach them equitably), but I'm learning that I
20. don't always know how to do that, and oftentimes I assume things about students
21. without even realizing that I'm doing it. We found out at this meeting that Amy has
22. Aspergers, which no one knew beforehand (75% of the school year has gone by
23. already!). I was amazed at how a handful of things that Amy's parents told us about
24. herself and her behaviors changed my view of her. After this meeting, I was thinking
25. back on interactions I've had with Amy where I would have treated her differently if I
26. would've known she had Aspergers. Things like Amy's problem turning in
27. homework, and her baffled, confused, nervous reaction when I ask her to get her
28. homework out. Previously, this situation looked like Amy had not done her
29. homework but was trying to cover it with the excuse that she couldn't find it over and
30. over, but in the meeting I learned that a legitimate behavior of Amy's is that she has
31. trouble remembering things like this and being consistent, and she becomes
32. uncomfortable and nervous with confrontational interactions like I've had with her. If
33. I would've known this, I would've reminded her about her homework at the beginning
34. of class and given her more time and space to turn it in, or I would've talked to her
35. one-on-one after class instead of in front of her table so she wouldn't be overwhelmed
36. with so many people around her. I've also noticed Amy drawing a lot during class
37. lately when she should be engaging in tasks with her team. I've been asking her to put
38. her notebook away, but she hasn't been responding very well. Through talking about
39. this issue in our meeting, I learned that Amy draws when she gets overwhelmed, and
40. it's been happening more since I put her at a table with four people instead of three. I
41. wish I would've taken the time to talk to Amy one-on-one so I could have found out
42. this information earlier and helped her succeed and enjoy my class! This meeting
43. opened my eyes to the fact that students' behavioral problems always have a source,
44. and seeking to understand what that source is makes it so much easier to address that
45. students' needs. I think teaching students' my expectations is also totally valid, but
46. when I notice that consistently reminding students of those expectations isn't
47. working, then I need to seek out that student and try to understand them.

The main thing that began to help Stella think differently about how she approaches her

students was observing other teachers asking questions (ll. 8-9). The other teachers asked Amy multiple questions about her learning, behavior, feelings, and how they could help her. Stella contrasts this approach with her own which consisted of "assuming" (l. 15) that she already understood Amy. As a result Stella recognizes the limitations of her abilities ("I don't always know how to do that" ll. 19-20; "without even realizing that I'm doing it" l. 21) despite her intent to "understand their individual needs" (l. 19). The things that she learned about Amy from her parents caused her to reevaluate past interactions. As she describes these previous situations she describes thinking from a deficit perspective about Amy ("Amy's problem" l. 26; "trying to cover it" l. 29; "she hasn't been responding very well" l. 38), but does not critique these assumptions.

While critiquing these assumptions could help Stella develop a more socially just treatment of her students, it would require that she point her flaws out to me. Stella does note that her choices made the problems worse and that she could have avoided these situations had she "taken the time to talk to Amy" (l. 41). She concludes by proposing a balance between "teaching students my expectations" (l. 45) and trying to "understand" (l. 47) her students. However, understanding comes only after there is a problem with her expectations (ll. 46-47), thus they are still given primacy. Stella's journal entries illustrate a back and forth between teacher-centered and student-centered thinking, where the teacher-centered remains dominant but is conceding to a more respectful approach to working with students.

Student-centered approaches to teaching are still fairly uncommon in secondary mathematics education (Ellis & Berry, 2005). Thus Stella's move in this direction feels like significant progress and I want to highlight it as I did in my responses to Stella's

journal as well as in my analysis here. Stella's emphasis on these points may also reflect a White, progressive idealization of education reforms, such as student-centered teaching. However, the fact that teaching becomes more "student-centered" does not mean that it also becomes more socially just. It is tempting to suggest that student-centered teaching address the equity dimension of identity. However, as Gutiérrez (2012c) explains, identity includes students drawing on their own resources and becoming better people by their own standards. There is no mention of that here. Rather, the standards are still Stella's (or the dominant discourses of mathematics education as represented by Stella), which are assumed to be universal (hence White) standards. Student-centered teaching may be necessary or at least fit better with an identity focus. However, by itself it is not enough.

Jane

Being in Charge

The transcript that follows is a selection from the conclusion of Jane's final report of her action research project. Jane worked in a small, diverse mathematics and science focused charter middle school. Her mentor teacher's class was built around a series of projects, student experiment, and student discussion. In this class students daily created extensive mathematical arguments, presented arguments to the class, and questioned each other's work. Jane become very proficient at this style of teaching and students played a large role in the class, even though the teacher was still very much in control. Similarly Jane's writing focuses on her students. Jane's action research project originally set out to vary the type of homework that students were assigned to see if more of her students would complete the homework. She framed this as an issue of access, arguing that not

completing homework held some of her students back and that they may not be able to complete it at home due to lack of interest or lack of resources. Along the way she serendipitously found that when students had time to work on homework in class, with access to teacher support, more of her students completed it successfully. The struggle that her report illustrates is one between her previous beliefs (deficit perspective) about students and her growing recognition of her responsibility to her students, which ultimately led her to question the relevance of homework.

1. At first, I do not believe my methods for improving homework completion were
2. necessarily in alignment with providing students with equitable access for completing
3. their homework. My initial purpose was more to increase students' intrinsic
4. motivation for doing their homework and therefore increase the class completion of
5. homework. The only problem with this approach is that the blame for failure to turn in
6. homework is placed predominantly on the students with less accountability from the
7. teacher. As I attempted to change the type of homework that I assigned but then did
8. not see improvement, I noticed that I began to become frustrated with the students with
9. what I perceived to be their interest in school. I then realized that as a teacher, I cannot
10. be placing all liability on the students (especially 7th grade students!), the majority of
11. the responsibility is on the teacher. Thus, it was incredibly eye-opening the day that I
12. happened to have extra time and gave them that time to work on their
13. homework and see the improvement. It was at this time that I realized that I was not
14. providing equitable access to students by trying to change their motivation, rather it
15. was my responsibility to ensure that all students have the resources necessary to know
16. that they can complete their homework.
17. However, I am unable to control their home circumstances. Therefore, I believe that
18. homework completion improved greatly when students were given time in class
19. because I am providing them with the resources that they need to at least begin their
20. homework. In this way, all students have access to some help before they head home
21. to their diverse, backgrounds. Even though giving students time in class to complete
22. homework seems like a menial change, I believe it makes has a colossal impact on
23. students. In fact, this seemingly insignificant change helped me to realize that too
24. often, positive change relies on teachers, not on students. In addition, even though we
25. cannot change everything about a student's life, there are some things which we can
26. control and should make every effort to provide students with as many resources as
27. possible.

In this paper Jane describes her change in perspective that helped her recognize her responsibility to her students, at least in regard to homework. By originally defining the

problem as one of student motivation (ll. 3-4) she was led to then blame her students (l. 5), which led to frustration directed towards her students (l. 8). She was clearly drawing on dominant discourses of individual responsibility, leading her to blame her students. In contrast, after her "eye-opening" (l. 11) moment she views herself as a resource for her students and responsible to them. The shift discursively happens in lines 11 through 13.

Prior to this section the discursive focus is on Jane, her actions, and her desires. She positions herself as the agent and in this sense is trying to control her students and their homework completion. Notice throughout this section the frequent use of "I" and "my," especially "my methods" (l. 1), "my initial purpose" (l. 3), and "I attempted . . . I assigned" (l. 7). She as the teacher/researcher is manipulating the variables in order to increase student motivation to complete homework. Beginning in line 11, though, the positive changes are made to appear to have happened by chance. This is done through the use of "happened to have" (l. 12) implying that neither she nor her students were responsible for the extra time. She then begins to focus less on her actions and more on what her responsibilities are. There is still a sense of control ("ensure that all students have . . ." l. 15), but the control is directed at herself and the classroom environment, rather than trying to control the students themselves.

Jane has in this way changed the focus of her authority, it is not an authority over her students, but authority over the material resources of her time and classroom time and what is done with that time. In this section she twice mentions giving time ("students were given time" l. 18; "giving student time" l. 21), which she refers to as "resources" (l. 19). In addition her support is another resource that these students have access to during this time. Finally as she closes she makes herself responsible ("relies on teachers" l. 24)

for those things which she "can control" (i.e., not her students, ll. 25-26). Jane reports a shift in her thinking about her students and understanding her responsibility to them, especially in terms of access. She places limits on how much teachers can change (l. 25) and by describing teacher control as over the resources (rather than students, l. 26) leaves open a position of authority for the students as well. By addressing access in this way Jane might reasonably expect a corresponding increase in achievement.

What seems to remain unseen beneath the surface of Jane's shift are the identity and power dimensions of Gutiérrez's (2012c) equity framework. The original focus of her project was to change the content of the homework she assigned. She wanted to see how homework completion changed when she assigned homework that was more relevant to her students' lives, was more accessible, and that had the potential to involve students' families. These kinds of assignments had the potential to address Gutiérrez's (2012c) critical access of identity and power. However, by framing it as an issue of homework completion and access (drawing on dominant discourses of individual responsibility), and because of the more dramatic results that came as she gave her students more time, the more critical aspects of her project dropped out.

Karl, Stella, and Jane each take a critical look at their use of authority in the classroom. Karl explicitly names authority and his discomfort with it. Stella describes her authoritative behaviors, the problems that resulted from her authority, and how she learned from others. Jane points out how she originally used her authority to place blame on her students, then how she used her authority to exercise control of the resources available to students. While each focused on authority in different and meaningful ways, none of them included student perspectives on the use of their authority or on the ways

that they might use authority differently. Each critiques her or his practice, but without including the perspectives of students and critical colleagues the potential effects of that critique are limited. This lack of student perspectives also hides the ways students of color viewed their teaching. Karl, Stella, and Jane have each, in different ways, picked up Gutiérrez (2012c) challenge to be in charge of the classroom. However, without a corresponding role for students they are left without the balancing tension of "not being in charge." Additionally none of them meaningfully addressed the way that their race and gender shaped their relationships with their students or their use of classroom authority.

Balancing Achievement and Identity

Lisa

An Achievement Focus Damages Identity

Like Stella, Karl, and Jane in the previous section Lisa also takes up the issue of being in charge. However, while Stella, Karl, and Jane were critical of their use of authority, Lisa positions herself as more authoritative in order to mitigate the damage she sees done by traditional measures of student achievement. Lisa taught in a small, diverse mathematics and science focused charter school. Her mentor teacher was an excellent model of reform teaching methods and of creating her own materials. She encouraged Lisa to also follow these practices. Lisa's mentor clearly focused on the content and was more traditionally authoritative. She would grudgingly accept social justice ideas as long as they did not take away from rigorous content. In these last two respects Lisa differed from her mentor. Lisa wanted to partner more with her students and placed greater value on social justice mathematics. Thus it seems that Lisa felt pressure to balance her mentor's expectations with her own teaching style and social justice commitments.

The difficulty in balancing these demands is reflected in Lisa's final paper on her action research project. Lisa's project focused on finding better ways to assess her students so that she could gauge their level of understanding and so prepare herself to teach them. While her chosen topic does not stand out as having a social justice focus, Lisa's explanation of her choice and reasoning draw heavily on equity arguments (some of which I have cut out to focus on her struggles). Her writing particularly illustrates her struggle with the potentially negative consequences of assessment on students and a desire to honor student identity and empower them mathematically. Prior to the selection below Lisa provides a detailed explanation of her teaching context and has just finished explaining the format and style of her tests.

83. My sense of unhappiness came from my perception of the aggrandized role of the
84. scores on these exams in my class. Though we allowed for improvement over a term
85. to have a positive effect on a students' grade, and we allowed for test corrections on
86. each midterm, the test scores were still devastating to some of my students' grades.
87. I am not able to change the set-up of the class—the percentages in the grade
88. breakdown were agreed upon by the entire math department, and I am working within
89. the environment these professionals (trustworthy and deeply caring professionals, at
90. that) have set up. In any case, I want my students to be able to perform at high levels
91. on these types of exams. I believe that the exams we created were full of
92. opportunities to demonstrate solid understanding of important mathematics. As part
93. of my goal as a teacher is to work with my students toward this understanding, I have
94. no qualms about giving my students tasks like the ones included in the summative
95. assessments I gave. Additionally, I want my students to become accustomed to
96. performing at high levels on challenging exams, so that they can succeed in academia
97. for several years to come. What I did want to change, however, was the absolute
98. nature of these exams—I see no reason why I should, or even have the right to, tell a
99. student that he or she deserves a specific score in my class, or that this score reflects
100. his or her knowledge or understanding of mathematics. I did not feel justified in
101. giving certain of my students the grades I gave them, especially because I felt that, if
102. my students demonstrated a lack of understanding on an exam, it had been my job as
103. the teacher to have tried enough different methods to allow that student access to
104. understanding, and that I had failed to accomplish this. Thus, the grade I was giving
105. should not have been final—obviously, if my student had not been allowed access to
106. understanding of a concept, my work with that student was not finished, and the
107. assessment would be a formative assessment only. All the summative assessments
108. appeared to do was to tell my students and me that my students had not achieved to

109. the required level. I assume that this makes students feel negatively toward my class,
110. specifically, and math and the progressive nature of learning math, generally.
111. What I hoped to achieve, through focusing on assessment for my action research
112. project, was to gain such an accurate idea of my students' understanding that I could
113. design engaging and appropriate lessons. In the process, I would be able to identify
114. misconceptions or holes in understanding before giving a summative assessment, so
115. that performance on these summative assessments would improve. I would also be
116. giving my students many opportunities to demonstrate their understanding, and
117. would be able to soften the blow of a low score on the summative assessment by
118. scoring these "opportunities" in such a way that the scores reflected my opinion of
119. what should go into a student's grade—their effort and willingness to try on a given
120. math task. The final byproduct of this action research project design would,
121. hopefully, be an improved sense of well being for me and my students, as everyone
122. could feel that their efforts were appropriately rewarded, that a lack of
123. understanding in spite of such effort would not be punished, and that the grade they
124. received in my class was deserved.
125. This is still very much a work in progress—I do not claim to have achieved the
126. aforementioned goals. In doing this project, though, I have gained clarity on my
127. goals as a teacher and how to obtain them. I hope that in doing this project I have
128. had a positive effect on the environment in my classroom.

Lisa begins this section by stating her dismay at the "devastating" (l. 86) effect test scores had on her students' grades. She continues by noting the constraints that she is working under. In particular she feels that she cannot work against the grade breakdown established by the mathematics department. She states this as an absolute, "I am not able to change the set-up of the class" (l. 87). But then later suggests that she is not entirely in disagreement with them ("In any case, I want my students . . ." l. 90). Unstated and unquestioned in this paper is the perceived need to assign grades based on tests at all. This represents an overlap of the discourses of her local context and the dominant discourses of mathematics education generally.

Important in this section are the desires that she expresses for her students. These desires include that her students "perform at high levels on these types of exams" (l. 91), "to work with my students toward this understanding" (l. 93), "become accustomed to performing at high levels on challenging exams" (ll. 95-96), and "succeed in academia

for several years to come" (ll. 96-97). All of these correspond to traditional measures of academic success and seem to include the tests that Lisa and Michelle (her mentor teacher) made, possibly standardized tests, and higher education. Lisa's statements reflect the standard discourses associated with these traditional measures of mathematical success. This focus could be framed as reflecting the achievement portion of Gutiérrez's (2012c) equity framework. From here though she explains the problems that she sees that are created by these exams and the expectations of grades. First among these is the "absolute nature" (ll. 97-98) of the scores associated with these exams. She links this to her telling students that she or he "deserves a specific score" (l. 99) and that this score matches the students "understanding of mathematics" (l. 100). These issues lead her to question her authority as a teacher ("I did not feel justified in giving certain of my students the grades I gave them" ll. 100-101).

Lisa does not, at this point, question the validity or accuracy of the tests (although she does elsewhere), but instead questions her own teaching. She felt the only useful information from the tests was that her "work with that student was not finished" (l. 106). She recognizes that this view of testing is inconsistent with using tests to assign grades (l. 107) and that assigning grades has negative effects on her students (ll. 109-110). These points indicate a concern with student identity, in particular in regard to students developing a sense of their mathematical capability. For Lisa, in this context there is a conflict between her desire to promote student achievement and working with her students to create positive mathematical identities. This conflict illustrates the challenge of addressing both the critical and dominant axis of equity, when the dominant maintains its emphasis on abstract mathematics. Additionally it shows the messiness of staying in

the conflicts between social justice and dominant discourse, without resolving them. Too often our attempts to resolve these conflicts lead us back into the dominant discourses.

As Lisa continues she begins to explain her hope to reconcile both her context (where the grading scheme is fixed) and her own desires that her students achieve well on these tests with her questioning of her authority to assign grades and the negative effects of these grades. In this process she is taking on the significant tasks of using assessment to better prepare her teaching (l. 113), and to better understand what her students need (ll. 113-114). Throughout this section she positions herself as the primary or only agent (note how frequently she uses I followed by a verb), which suggests that she sees herself taking control of the situation, despite the context and the norms of testing and grading. Additionally in order to accommodate her context and the discourse she redefines some of her students' scores to reflect "effort and willingness to try" (l. 119) in order to "soften the blow" (l. 117) of the scores where she feels she has less control. While Lisa begins with her lack of control of the context, she closes by taking control, positioning herself as agent, and redefining terms to meet her own and what she perceives as her students' needs.

Developing Student Identity

In the next section Lisa turns her attention more explicitly to her students and the hopes that she has for their identity development. Her perspective shows an important respect for her students and their identities.

Lines 129-133 (Lisa summarizes Gutiérrez, 2012c equity framework)
134. My goal, in part, in being a teacher was stated above: to work with my students
135. toward understanding in mathematics. The unstated portion of this is the reason
136. behind this work—I want my students to become proficient researchers and
137. perseverant problem solvers, so that they can look at what takes place around them

138. with a critical eye, and can work toward change and justice for themselves and for
139. others. This practice can become strong in a math class, where perseverance and the
140. willingness to try are required for success. As well, math can be used as a tool to be
141. used in this work—my students should be able to encounter a spreadsheet of
142. seemingly incomprehensible data with no fear at all. Any work I do in my class to
143. engender positive attitudes toward math and education, to create opportunities for
144. my students to hone their skills, and to encourage my students to strengthen their
145. reasons for achieving in math is done with this at the forefront of my mind.
Lines 146-156 ((Lisa describes some of her formative assessments))
157. In order to achieve in math and to go on from my class to gain entrance to and
158. achieve in higher education, to practice ideals and ideas and to become powerful
159. citizens, I need my students to have a pretty good attitude toward math, about their
160. ability in math, and about education in general. One of my issues was what I
161. perceived to be detrimental actions against my students' relationships with
162. mathematics and education via summative test scores. My attempts to support
163. students in achieving higher test scores, by using formative assessment to inform my
164. design of the correct tasks for each student, and my attempts to cushion the blow of
165. a low exam score, by reinforcing student effort and progress toward understanding,
166. in both attitudes and grades, were made with the hope of encouraging my students to
167. achieve in mathematics, and in school. This is a very long-term goal, and one that
168. will likely take many years of practice. Working to engender positive attitudes
169. toward math while providing students with challenging and important math tasks is
170. a tricky balance. My hope is that, although the education system is not perfect, my
171. students will have a full picture of the positive things they can achieve by
172. persevering in school.
Lines 173-180 ((Lisa describes her assessments))
180. Again, my process was far from
181. perfect, but, keeping these goals in mind, I believe that I will come closer to making
182. my classroom a place where the identity of each student need not be shed as a way
183. to conform with the teacher's ideas of what success in a math class looks like.

Lisa's goal statement (ll. 134-135) is an important one and reveals her focus on and respect for students. The phrase "work with" (l. 134) implies a relationship of mutual respect in which the teacher and students work together. Contrast this phrase with other possibilities including "teach," "instruct," or even "empower" all of which imply a teacher-centered, doing-to as opposed to working with. Secondly in her statement is the emphasis on "understanding in mathematics" (l. 135) as opposed to scores or grades. The emphasis on understanding is potentially better suited to aid students in future success in mathematics classes.

From here Lisa elaborates on the specific outcomes she hopes to have for her students. She names them "proficient researchers and perseverant problem solvers" (ll. 136-137), which will lead them to have a "critical eye" (l. 138) and the ability to work for "change and justice" (l. 138). Each of these descriptions references student attributes that she believes can be developed in the mathematics class, but the mathematics is positioned as secondary to these student attributes (note the next few lines ll. 139-141). By placing her students and student attributes first Lisa gives primary importance to student identity. As she continues she cites her desires to work to "engender positive attitudes" (l. 143) and to "strengthen their reasons for achieving in mathematics" (ll. 144-145). These also support a focus on student identity, especially the emphasis on her students' "reasons for achieving" as opposed to the teacher's or the school's reasons. In this way Lisa alternates between an achievement and an identity focus, with neither fitting well with the other.

As Lisa continues she turns her focus to preparing her students for college and future mathematics classes. This creates a conflict between her student identity focused ideals just described and the score/grade focused discourses of college admission and tests. Lisa sets up this contrast from the beginning noting the need for achievement "to gain entrance to and achieve in higher education" (ll. 157-158) which can lead to their becoming "powerful citizens" (ll. 158-159). The conflict between an identity focus and test score focus is summarized in the next sentence, which points to the "detrimental actions against my students' relationships with mathematics and education" (ll. 161-162). Throughout this report Lisa has positioned herself as an agent, in control of and responsible for what happens in her class. However, the passive construction of this sentence removes responsibility leaving only the "summative test scores" (l. 162) as the

cause of the "detrimental actions." This leads to a lengthy delineation of her "attempts to support students" (ll. 162-163), which she does take responsibility for. She ties these efforts to "encouraging my students to achieve in mathematics" (ll. 166-167). Thus, at this point, student identity and understanding have been subordinated to achievement. However, she seems to turn again by noting her hope that her "students will have a full picture of the positive things they can achieve" (l. 171), which returns to an identity focused view. As she closes this section Lisa reemphasizes her goal of honoring student identity in balance with her own views of success (ll. 182-183). The conflict between identity and achievement is largely a result of the dominant discourses of abstract mathematics that is emphasized in schools with the accompanying focus on right and wrong answers. This kind of judgment makes meaningfully addressing student identity more difficult.

Despite her efforts Lisa is still largely unsatisfied with the results of her students' test scores. In the section that follows she reasserts her responsibility to her students and describes how she helps to mitigate the effects of these tests.

Lines 184-220 ((Lisa describes the methods of her action research project))
220. The summative test scores that followed, though, left me very discouraged. This is
221. when the second goal of my action research became more solidified. I felt wrong
222. giving some of my students the scores they had "earned" on this midterm—I knew
223. that I had failed these students. To remedy this, I made it known to my students that
224. improvement from one summative assessment to the next would be greatly
225. rewarded, and that I would endeavor to spend as much time in- and outside of class
226. to make sure that students that were willing received the support they would need to
227. see this improvement.
Lines 228-261 ((Lisa describes a group assessment and a formative assessment she gave her students))
262. However, my students did seem to benefit, to some extent, from my work with
263. formative assessment over those few weeks. All of this culminated in the final
264. summative assessment for the third quarter. I designed an exam similar to those we
265. had previously given. It contained a variety of types of problems that fell into a
266. variety of difficulty levels. I do believe I was able to affect some degree of positive

267. change in my classroom with my action research project, given the results of the
268. exam. Though my preparation, implementation, and analysis of the formative
269. assessments I had given were not perfectly effective, and I still had much work to
270. accomplish, I was able to see an improvement on this summative assessment. Some
271. of the students who I had agonized about giving scores of less than 20% on the
272. midterm were able to improve so much that their final score in the 40%-weighted,
273. individual, summative assessment category was high enough to pass the class with
274. ease.

Lisa again describes her discomfort with her students' test scores (l. 220) and accepts responsibility for these scores. She goes so far as to claim "I had failed these students" (l. 223). She also notes that she "felt wrong" (l. 221) giving her students the scores that she did, but rather than try to justify her feelings she attempted to "remedy this" (l. 223). This attempt demonstrates her commitment to her students and their feelings about themselves and about mathematics and the struggle in working towards high-achievement as well as honoring student identity.

Despite this commitment and her emphasis on student understanding and identity stated previously, this next section shows her continued reliance on summative assessments to gauge both her students' understanding and her own efforts. Just as her department places greater weight on these assessments in determining a students' grade, she places greater weight on them in measuring her and her students' success. Note the prominence that she gives to the assessment, stating that "all of this [previous work] culminated" (l. 263) in that single exam. However, this prominence given to test scores is also related to her concern about the negative effects of tests on students' grades and through their grades on their beliefs about their own capability and worth. After describing the exam and the results she then turns to her concern about grades. She notes the improvement of some of her students (l. 272) and that, with the exam results, some of the students she was concerned about passed the class (l. 273). Thus while her teaching

context encourages and dominant discourses push her to (perhaps) an overreliance on summative assessments to measure her effectiveness she mixes this in with her concern about the effect on her students and so strikes this balance between her context and her students' needs.

With her concern about student grades satisfactorily resolved, Lisa turns more of her attention to meeting her students' needs and attempting to match the curriculum to those needs and their interests.

275. Thus, I became stronger in my conviction that frequent and varied formative
276. assessment was key to the achievement of my goals. I started to collect written work
277. from my students daily, and to analyze it to help me create tasks that were more
278. appropriate and engaging for students, even differentiating tasks by providing
279. extensions for students who had shown proficiency with a certain concept while
280. providing a new avenue to the same concept for students who had not benefitted
281. from my previous task. I am still working on making it known to my students that
282. their effort and willingness to attack difficult problems is far more important to me
283. than the score of their summative assessment, but I think I have made progress
284. there, too.
285. Wishing to continue supporting an environment where students are the creators of
286. mathematics, and to continue to collect formative data, I asked other open-ended
287. questions as we proceeded into our statistics unit. I incorporated some team
288. activities and some self-evaluations, as well, but my favorite were the open-ended
289. questions. A particular favorite was one dealing with realistic data populations and
290. whether or not they could be normal. I didn't know the answers to my questions, I
291. just wanted my students to create arguments using some of the features of the
292. normal distribution, in context. I felt really good about this assessment. I felt that my
293. students were able to contribute to the discussion, and that there was room in the
294. assessment for more than one correct answer. It seemed, to me, that their
295. understanding of the content was strong and intuitive. I benefitted from this
296. assessment, as well, beyond getting what I judged to be an accurate picture of their
297. understanding. Not knowing the answer to a question that I posed was refreshing—
298. students were able to surprise me, and they devised strong arguments that I was
299. more receptive to, not having come into the discussion with one set path, or one set
300. answer, in my mind. I would do well to keep that in mind as I progress.
Lines 301-307 ((Lisa describes her journal keeping during this time))

Lisa continues to emphasize assessment, finding them "key to the achievement of my goals." However, she now provides more varied types of assessment (ll. 276, 279-280)

and uses assessment to engage her students (l. 277). In this way she is attempting to blend the need she feels to assess her students (both to meet department/societal expectations and to better understand her students) with her desire to respect and engage them. This blending does not always come together smoothly as she notes the difficulty of showing students that "their effort and willingness" (l. 282) is more important than their scores. As she continues Lisa emphasizes the importance of "open-ended questions" (ll. 286, 288-289) as these provided opportunities for her students to be "creators of mathematics" (ll. 285-286). This perspective creates a different role for both student and teacher. Where known-answer questions portray teaching as knowledge transmission (with teacher as knower and students as empty recipients) Lisa's perspective shows learning as a creative process. This process recognizes the student as agent (creator of mathematics) who can "create" (l. 291), "contribute" (l. 293), "surprise" (l. 298), and devise (l. 298). This orientation led Lisa to be "more receptive to" (l. 299) her students and their ideas. While not stated, this implies that she is not always receptive to their ideas and part of the struggle in this area may be the restrictions she feels from more traditional assessments and how they represent mathematics in ways that limit student contributions. Importantly these questions also addressed "realistic data populations" (l. 289) and so were a break from the more abstract mathematics she had focused on in previous assessments. In contrast to some of her previous statements about feeling wrong about giving students grades, this time she "felt really good" (l. 292).

Building on this she describes her next experience as "delight" (l. 308). This comes from an "alternative summative assessment" (ll. 308-309) on statistics. The choice of "delight" is not commonly used to describe a mathematics class and much less so an

assessment. This suggest the possibility of Lisa approaching this task playfully, in particular an openness to surprise. Perhaps the use of "alternative assessments" and the nonstandard, less abstract content of statistics[22] created room for her to play. The emphasis throughout this section is on student choice and agency, but this only happens within the context of an "alternative" assessment. By designing an alternative assessment Lisa seems to feel more freedom to work with her students, but naming it alternative positions this kind of work as marginal and occasional rather than as the norm within a class.

308. The pinnacle of my project has been my delight with an alternative summative
309. assessment that Michelle and I designed together. We asked students to use
310. everything they had learned during our statistics unit to compare two populations
311. using some variable that they felt measured an injustice. Students were free to
312. choose any topic that interested them, to collect data using the method of their
313. choice, to represent their data however they wanted, and to write about their
314. results—what they found, what it meant in context, and whether or not it supported
315. their hypothesis—until they felt their argument was complete. This project, to me,
316. represented a success in almost every way—my students demonstrated a strong
317. understanding of mathematical processes and a developed ability to obtain
318. information, judge its reliability, and analyze it with a goal of learning more about
319. something that they found unfair about the world. I got to read papers about women
320. not being allowed to drive in Saudi Arabia, salary gaps between men and women,
321. college enrollment disparities between Latinos and Whites, double standards
322. between men and women, HIV infection rate disparities between Native Americans
323. and whites, and many more impressively interesting topics. I had my students
324. present their work to the class for a few extra credit points, if they so chose, which
325. many did with confidence and grace. Their peers asked important, discussable
326. questions, and I was able to see my students in a new light. There is still much work
327. to be done, but I felt that my students showed me that they could achieve within
328. Gutiérrez's framework.

Conclusion

329. As mentioned, my action research project is still in progress. I need to hone my
330. skills in designing, implementing, and analyzing formative assessments to
331. accomplish my goal of using the information gained from effective formative

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[22] Traditionally statistics has been a very small part (if included at all) of mathematics curricula. Statistics is less abstract in the sense that it is much more frequently connected to data drawn from real-world contexts.

assessments to design appropriate and engaging tasks for my students. This is of the utmost importance, as I require my students to be interested enough in math and other parts of their education to persevere in problem solving and to achieve at high levels throughout their continuing education. I want my students to feel that they contribute and create the math in my classroom, so that student identity remains strong. I feel that my second goal, to be able to feel justified in the grades that I give my students, was only partially accomplished this year. I do feel that my students have more of a sense that their improvement and continued learning is most important than they did at the beginning of my tenure. Until I can figure out a grading system that succeeds to reward effort and perseverance, and until I become effective enough with formative assessment to consistently feel that any summative assessment I do give my students was perfectly appropriate, I will continue to work on this part of my goal.
The strongest ideas I have solidified through this process were the importance of reflection on one's teaching practice, the significance of Gutiérrez's framework for equity in education, the necessity of allowing students to have a voice in the classroom, and the need for continued learning in this profession. I will keep working!

In this section Lisa describes her students as "free to choose" (ll. 311-312), "using the method of their choice" (ll. 312-313), "however they wanted" (l. 313), and to continue "until they felt their argument was complete" (l. 315). The lengthy list of topics (ll. 319-323) that students presented also emphasizes this point. Lisa evaluates this assessment as "a success in almost every way" (l. 316) and points to her students' "understanding of mathematical processes" (l. 317) as well as their development of various abilities in relation to "something that they found unfair" (l. 319). Thus she was able to blend her desire for her students to develop mathematical understanding with what she sees as her students' interests and concerns. This leads her to "see [her] students in a new light" (l. 326), specifically that they "could achieve" (l. 327).

As she concludes Lisa recognizes that the challenges she faces are not resolved they are "still in progress" (l. 329). She points out her role in creating "engaging tasks for [her] students" (l. 332) which acknowledges both her responsibility and the importance of students' needs and interests. At this point the "effective formative assessments" (ll. 331-

332) become a tool to help her meet her responsibility. This responsibility is "of the utmost importance" (ll. 332-333) because she links it to their need to "persevere" (l. 334), "achieve" (l. 334), "contribute" (l. 336), and "create" (l. 336). All of which recognize both the agency and ability that her students have and all with the end that "student identity remains strong" (ll. 336-337). The verb choice "remains" is an important one. This implies that she believes students already have a strong identity and that identity is valid and valuable in her eyes. The use of remain implies that mathematics is potentially damaging to students' identities and that without the steps that Lisa took to provide students with opportunities to "contribute and create the math" then their identities could have been damaged. She has clearly positioned these concerns and her responsibility to address them as primary, which leaves her "second goal" (l. 337) "to feel justified in the grades that I give" (l. 337) subordinated. This struggle to bring together the departmental/discursive norms of summative assessment with her responsibility to student identity and agency is a more difficult one. Thus she pledges to keep working on this aspect of her teaching.

The section presented above is the conclusion of her report. It is both significant and important that she closed by pointing to the incompleteness of her work. She notes "the need for continued learning in this profession. I will keep working!" (ll. 348-349). In this way she shows her embrace of the uncertainty of this work; she refuses to present herself as having figured out how to resolve the issue that was her stated goal. This sense of incompleteness is woven throughout her report. She states that her work "was not finished" (l. 106), not yet achieved (ll. 125-126), would require "many years of practice" (ll. 167-168), yet to be accomplished (ll. 269-270), in progress (l. 281), yet "to be done"

(ll. 326-327), "in progress" again (l. 329), and as requiring continued work (ll. 343-344). Her willingness to present her work as incomplete reflects a humility that Warren (2001) cites as necessary in order to do Whiteness differently. Her willingness to continually highlight her own shortcomings and the sometimes detrimental effects of her actions also reflect the humility and critique that Applebaum (2010) suggests we need in order to engage in social justice work. Like Karl, Stella, Jane, and I there is no discussion in her writing of how race affects her efforts and how her students receive them.

Normative Goodness and Fitting In

Esperanza

Competing Ideals

While Lisa struggled to balance her ideals in the classroom, Esperanza struggled to balance the competing demands she felt as a Latina mathematics teacher. Esperanza taught in a large, diverse public high school. Her mentor teacher was open to and occasionally used reform methods, but he was the only one to do so in the department, which attempted to dictate when topics should be taught. Esperanza's mentor encouraged her in exploring ideas and different approaches to teaching. However, he generally took a "hands off" approach when students were struggling. In contrast Esperanza wanted to get involved. She would talk to students to learn how she could support them, then also go to other teachers or the principal (against her mentor's recommendation) in order to access greater support for her students. These contrasting styles likely contributed to the conflicts she describes in her journals. As we turn to examine some of her struggles note how open she is about her feelings and the challenges that she faces. Throughout her writing Esperanza is clear about the conflicts she feels between what she wants to do

based on her community commitments and what she feels she has to do as a mathematics teacher. The journal entry that follows was written during the first week of the semester, following the first class.

Reflection 1
1. Another thing that was mentioned in class was about how do we make mathematics
2. relevant to students? Definitely this is something I often think about because I do not
3. know how to make my lessons relevant to my student life. I feel that in order to know
4. what my students care about, from the beginning of the school year I have to teach
5. them to communicate to me sincerely what is it that they would like to see in my math
6. class. Yet, I am unsure on how I would accomplish making it relevant, certainly this is
7. something I would like to explore in this class, whether it is through the action
8. research project (if we can consider this as a topic) or through class discussions.
9. Last, the papers we wrote about this week made me think back to how and why I
10. decided to be a mathematics teacher. Certainly, this brought back the ideas and goals
11. I had when I decided to pursue this profession, unfortunately I sometimes forget
12. about my ideas when I am asked to do things differently in the school. At times it
13. frustrates me and I feel very incapable of succeeding, yet been able to write up my
14. beliefs for becoming a math teacher once again, allows me to remind myself why I
15. chose this career. And, somehow it makes me stronger to employ those beliefs in my
16. class.

From the very beginning Esperanza makes her desires to "make mathematics relevant to students" (ll. 1-2) clear, but she does not know how (ll. 2-3). She has a sense that she should learn from her students what they want (l. 5), but then is unsure what to do with that information once she has it (l. 6). Notice here that Esperanza is not concerned with her mathematical knowledge, but rather with her lack of knowledge of how to make that mathematics relevant and to match it with her students' interests. She also points out that in her student-teaching she is "asked to do things differently" (l. 12) than what her own ideas and goals are. This focus causes her to "forget about my ideas" (ll. 11-12) and illustrates the dominance of traditional mathematics education over her ideals, including making the mathematics relevant to her students. The conflict posed by mathematics and school expectations are enough that Esperanza sometimes feels "incapable of

succeeding" (l. 13), but she draws strength from remembering her goals. It is clear that she feels pulled in multiple directions in terms of what her goals were and what she feels she has to do in order to be successful.

Multiple Discourses and Fitting In

As Esperanza continues to write she spends more time exploring some of the conflicts that she feels. I have grouped the next two journal entries (from weeks 5 and 6) into a single transcript since the second entry (week 6) is in some ways a response to some questions I asked after her first entry (week 5). Both my questions and her responses are included in what follows. These entries follow readings and a class discussion about discourses, especially on discourses that influence mathematics education. In this section Esperanza explains some of the multiple discourses that she relates with and how they conflict. They also clarify some of the conflicts that she mentioned in her entry above. My initial questions are in response to her previous journal entry (not included). In that journal Esperanza described multiple discourses as influencing her. Principal among these were a community discourse and a mathematics teacher discourse. While Esperanza did not mention conflict, it was clear that she felt that bringing the discourses together were problematic for her. Of all of the teacher candidates Esperanza was the only one to identify a conflict between who she felt she was and the dominant discourses of mathematics. This may arise from her own feeling of being an "outsider within" (Collins, 1986) where as a Latina she is seen as outside the dominant construction of a mathematics teacher yet has worked to gain acceptance within this community. Additionally it reflects the discursive pressure to conform that women and students of color may feel in order to advance in mathematics (Gutiérrez, 2012b).

These conflicts led to my questions about discourses (l. 2) that frames this journal entry. Notice here the conflict between what she wants to do and what she feels she must do.

1. Teacher: You've mentioned several Discourses that influence you as a math teacher.
2. What do you do when these Discourses come in conflict?
Reflection 5
3. Esperanza: In my last reflection, I mentioned several Discourses that influence me as
4. a math teacher. And, indeed not all of these Discourses agree with one another all the
5. time. Often times, I have a difficulty understanding why has our school systems failed
6. many minority students. My Latin@ community Discourse often encourages me to
7. "fight" for equity in our minority communities in the US and it builds strongly believes
8. in me about why it is so important to question the system. Yet, my math teacher
9. Discourse often worries about teaching math equally to all students without the
10. concern that equality is not the same as equity. When my Discourses come in
11. conflict, I often find myself confused and unhappy about what I am doing as a
12. teacher. But, then I take the time to think about what my beliefs are and the reasons to
13. why I decided to be an educator. It is from there, that I tried to sort out the conflicts
14. that I encounter. Yet, sometimes it is difficult to pursue my beliefs when the
15. Discourse that I am in does not build on those beliefs. I want to be part of the math
16. teacher discourse and I want to "fit in," and therefore I do not always question the
17. ideas within this Discourse. I feel like I have no voice in the ideas they bring to me
18. and their dominant ideas conquer what I think.
Lines 19-37 ((Esperanza discusses some of the class readings and changes to make education more equitable))

As Esperanza begins, she explains the conflict she feels between two of her principal discourses, the "Latin@ community Discourse" (l. 6) and the "mathematics teacher Discourse" (ll. 8-9). The former pushes her to "'fight' for equity" (l. 7), while the latter promotes "equality" (l. 10) over equity. The conflict between these two cause her to feel "confused and unhappy" (l. 11). As she faces these conflicts she reminds herself why she "decided to be an educator" (l. 13). Despite her commitments she finds it "difficult to pursue [her] beliefs" (l. 14) and that she feels like she has to "'fit in'" (l. 16) the mathematics teacher discourse. When she tries to fit in she feels she has "no voice" (l. 17) and her thoughts are conquered by "dominant ideas" (l. 18). While her community

discourse pushes her to "question the system" (l. 8) in order to fit in she does "not always question" (l. 16).

As she writes about this conflict she associates the community discourse as "my beliefs" (l. 14), her "voice" (l. 17), and "what I think" (l. 18). This positions Latin@ community discourse as her priority and as part of her, whereas the mathematics teacher discourse is foreign and external. This externality is clear as she notes her desire to "be part" (l. 15) of it, "the ideas they bring" (l. 17), and "their dominant ideas" (l. 18) all of which suggest that the mathematics teacher discourse is outside of her and she is trying to get in. This is analogous to the situation Gutiérrez (2012b) describes for women and students of color who feel they have to leave part of themselves behind in pursuing mathematical goals. The struggle she feels is to "fit in" to the mathematics teacher discourse (in order to pass, get a job, etc.) while keeping her community commitments intact (and her sense of self). The mathematics teacher discourse places a normative pressure on her to leave behind those goals and ideals that do not fit what is understood as a "good" mathematics teacher. This apparent incompatibility of her community discourse with the mathematics teacher discourse causes the conflict that she feels.

Conflict

In my response to this entry I focus on her discussion of trying to fit in and ask her to explain more about this. Esperanza's response clarifies the conflicts that she feels between her ideals and what is expected of her as a mathematics teacher.

38. Teacher: Good thoughts. Where/when do you notice yourself trying to fit in
39. mathematically? Are there ever times when you do things to fit in even though none
40. of your peers (mentor teacher, classmates, etc.) are watching?
Reflection 6
41. In last week's reflection, I talked about how as I try to "fit in" to the Discourses I

42. avoid questioning the Discourse ideals. This tends to happen when I am new to a
43. discourse or when I do not want to feel different. This has become very problematic
44. to me, because as much as I do not want to forget where I come from an who I am, I
45. tend to do that to not feel singled out. When no one is watching, I do not feel that
46. need to " fit in," but sometimes the ideals or the ways we do things in a particular
47. discourse have been embraced by me so strongly that as a result I do things to fit in
48. without thinking about them. I am confused!... I have never questioned this before
49. because I thought that it was the way to do things here. So many times I have felt
50. different that if I can avoid it I will. Yet, for some reason I am more careful now days.
51. I figured that it has to do with the fact that I am reaching places where not many get
52. to be. I feel privileged (to be where I am because of the education I received) that it
53. is my responsibility to voice for people in my communities and others that do not
54. have the same opportunities I once had. And, this is where my Discourses come into
55. conflict with one another. I wonder if this only happens to people like me (minorities,
56. first generation students, second language speakers, etc.). Or, does it happen to
57. everybody in different forms? What do they experience and how do they feel about
58. it? Something that stuck with me from the reading this week is that Action Research
59. does not start with a question, but with a change in mind that will improve our
60. students learning. I am thinking of a possible topic about "making students see that
61. math is more than just algorithms." But I would also like to explore "ways to make
62. the curriculum more relevant to the community of students I am serving."

What is striking in these opening lines is the language of avoidance that Esperanza uses in order to fit into the mathematics teacher discourse. All of these occur in order to "not . . . feel different" (l. 43). This avoidance begins with "avoid[ing] questioning the Discourse ideals" (l. 42), not wanting to "feel different" (l. 43), not wanting "to forget where [she] come[s] from" (l. 44), and "to not feel singled out" (l. 45). She is "confused" and questioning (l. 48) and now she is "careful" (l. 50) and "avoid[s]" (l. 50) feeling different. While she never mentions what she feels different from, the implication is that, because she identifies as Latina (ll. 55-56) with her accompanying commitments to social justice (ll. 61-62), she feels different from other mathematics teachers.

There is some normative ideal of a "good" mathematics teacher that does not leave room for her to be Latina or to engage in social justice mathematics. She feels that in order to be a "good" mathematics teacher she has to conform to this ideal. The fear of

being caught not fitting in pressure her to avoid actions that will make her stand out. The influence has been so powerful that even "when no one is watching" (l. 45) she continues to follow the norms "without thinking about them" (l. 48). She mentions three times the desire to avoid feeling different (ll. 43, 45, 50). This desire conflicts with her "responsibility to voice for people in my communities" (l. 53) that she also feels as a result of the position of relative privilege that she holds. Connecting these conflicts with the privileges (l. 52) that she holds points to the precariousness of her position (outsider within). She recognizes that as a Latina mathematics teacher she is already different from the ideal of a "good" mathematics teacher and has to work in order to maintain her image as a mathematics teacher. Trying to fit into that ideal causes the "conflict" (l. 55) that she feels. This is illustrated in the two possibilities that she presents as possible topics for her action research project (ll. 60-62). The first fits better with a traditional mathematics education focus, while the latter is better connected to her community commitments.

Esperanza would struggle with these conflicts throughout the semester. At the end of this entry she mentions wanting to build a project around making her curriculum more relevant to her students (ll. 61-62). Ultimately she decided against this option, because of the constraints she felt in her student teaching placement. However, this appears to have been a strategic choice, rather than a capitulation, based on the context of her student-teaching as well as her position as student-teacher.

Conclusion

The purpose here is not to point out all of the areas where we have fallen short. Of course we have fallen short; teaching mathematics for social justice is a monumental task. Rather if we are to take on the challenge of Applebaum's (2010) vigilance or Gutiérrez's

(2009) equity stance we will engage teaching mathematics for social justice, knowing that we will often fall short and engaging anyway. This is, in part, what it means to work against dominant discourses. Each of us in different ways shows some of the difficulty of understanding and engaging in teaching mathematics for social justice. The selections from my journal show the challenges I feel in presenting a radical critique of mathematics education within a dominant mathematics discourses that discourages social justice and more radical perspectives. Karl's work illustrates the difficulty of calling out teacher authority and increasing student voice, when students are consistently positioned as passive and as mathematical novices. Stella shows the seduction of progressive ideals such as student-centered teaching that lure us into thinking that we are working towards social justice, while maintaining a central focus on abstract mathematical content. Jane's efforts show the challenge of balancing teacher responsibility to students with an active role for students in their education. Lisa agonizes over the difficulty of focusing on achievement while addressing identity within the context of school mathematics, without trying to resolve the tensions. Esperanza's heartfelt journal entries illustrate the normative effects of teacher preparation and the challenge of holding on to both ideals and identity in this process, especially for a teacher of color.

CHAPTER 8

CONCLUSION

Mathematics teachers who understand *why* social justice in mathematics is necessary, still may find what it *means* and what it *looks* like to be incomprehensible. I argue that this incomprehensibility stems, in large part, from the dominant discourses of mathematics, particularly school mathematics. These dominant discourses center on the insistent abstractness of mathematics and the binary of pure/applied mathematics. These discourses overlap with key discourses of Whiteness (such as normative constructions of "goodness," judge, or individual responsibility) in ways that reinforce the constraining power of each. In our efforts to understand teaching mathematics for social justice these dominant discourses played a significant role in restricting our ideas of how to teach for social justice.

Learning to work against and disrupt these discourses is not linear, nor can I say that now we "get it" (Gutiérrez, 2015; Thompson, 2003). Instead, in an attempt to disrupt the idea that we progress from not getting social justice to getting social justice, I have purposely not presented the transcripts in the previous chapters chronologically. Throughout the semester, both at the beginning, at the end and in between, our discussions moved in and out of the dominant discourses. Thus while our understanding of teaching mathematics for social justice is deeper than it was, we still find ourselves pulled into the dominant discourses. What follow are the conclusions I draw from the

previous three chapters in response to each of my original research questions. There will be a section for each question, followed by implications for mathematics education and mathematics teacher education.

The Discourses We Use

The first of my research questions is: What discourses do secondary mathematics teacher candidates invoke when discussing social justice in their own teaching practice? As we regularly discussed our own teaching and social justice we were often constrained by the dominant discourses of mathematics. Central to these discourses is the idea of mathematics as naturally abstract—that abstraction was a necessary condition in order to qualify something as mathematics. The discourses of mathematics as abstract were so familiar and comfortable for us that conceiving of a mathematics that was not abstract (or at minimum an applied version of an abstract concept) was challenging. These dominant discourses shape the relationships we establish with students, our understanding of what it means to be a good mathematics teacher, and what we recognize as school mathematics.

Relationships With Students

An overarching discourse of school mathematics is a discourse of correctness. This discourse arises from and supports the discourses of abstract mathematics and positions mathematics teachers as judges of the correctness of students' responses. While all teachers are similarly positioned to a greater or lesser extent, within mathematics education the judgments and evaluations are greatly narrowed by the focus on a single correct answer, or, at best, on following a correct procedure. This focus sets up

relationships between students and teachers that are both authoritative and defined by mathematics. Teachers and students (with few exceptions) cannot connect through the mathematical content, because the focus is almost entirely on correctness.

As we use these discourses we create and maintain a particular way of understanding mathematics, a particular way of being a mathematics teacher, and set up particular relationships with our students. While these discourses enable us to become mathematics teachers (and enabled my students to find jobs as mathematics teachers), they also narrow our ability to understand mathematics and mathematics teaching to the confines of these discourses. These discourses largely position mathematics teachers as authoritative and students as passive recipients of teaching (Skovsmose & Valero, 2001). There is no room for students to make mathematical contributions, although they may make other contributions, if they are sanctioned by the teacher. Further these discourses serve to divide students into groups with students who are mathematically capable perceived as more intelligent.

The "Good" Mathematics Teacher

The ideal of the "good" mathematics teacher was directly connected to a teacher's knowledge of and use of abstract mathematics. Discourses that continue to center abstract mathematics (such as constructivism or student-centered teaching) appeal to progressive teachers precisely because they maintain the link to our idea of a "good" teacher while seeming to respond to the flaws of more traditional mathematics teaching. However, their continued focus on the dominant discourses of school mathematics does not allow them to disrupt the divisions and exclusions created by these discourses.

Control is an integral part of discourses of being a "good" teacher. This control

over students and class content is manifest in multiple ways including the common IRE/F cycle. The evaluation portion of this cycle is encouraged by the discourse of correctness and by the apparent necessity for mathematics teachers to police what counts as mathematical. The ability of a mathematics teacher to control a class through evaluation is based on the teacher's knowledge of abstract mathematics. The discourses of standards and standardized testing are extensions of the idea of evaluation. These discourses emphasize measureable outcomes and position teachers as responsible for what their students learn (or do not learn), placing enormous pressure on teachers to conform to these standards, and to be more controlling in response to this pressure. All of these judgments and evaluations are perceived as certain because they rest on the certainty of abstract mathematics.

Determining What Counts as School Mathematics

The discourses of dominant mathematics position school mathematics as preparation for more advanced mathematics. This positioning requires school mathematics to emphasize the abstract and to privilege "pure" mathematics over applied. As a result when we tried to find alternatives to school mathematics in trying to get away from pure mathematics we turned to applied mathematics. However, because applied mathematics is part of the pure/applied binary (in which applied mathematics takes a pure mathematical concept and uses it in a specific context), our efforts usually led us back to pure mathematics. These discourses, by positioning mathematics as naturally abstract, disallow radical perspectives that view mathematics as a human creation or for the need for social justice within mathematics education.

The struggle we faced though was not just that these dominant discourses

disallowed social justice perspectives. Rather we find some aspects of abstract mathematics appealing. One of the appealing justifications for abstract mathematics is the discourse of mathematics as a form of communication. This discourse positions the abstraction of mathematics as both necessary and desirable in order to facilitate (universal) communication. In this sense mathematics as communication appears to become a tool to promote equality and democracy (Skovsmose & Valero, 2001), because of its apparent ability to cut across cultures and languages. According to this discourse, this universal communication can only occur through abstract mathematics. From this perspective, the potential merits of any other form of mathematics were always subsumed by the apparent necessity of communicating these mathematical ideas to others. This discourse held even when we were clearly able to understand that abstract mathematics is, in fact, not a universal form of communication and that there are a relatively small group who can use abstract mathematics as a means of communication.

Excluding Mathematics for Social Justice

The second question I asked is: How do the discourses secondary mathematics teacher candidates use around school mathematics in the United States interact with the discourses they use around social justice mathematics? By and large the discourses that we used around abstract mathematics and mathematics education inhibited our ability to understand what it means to teach mathematics for social justice. This occurs by defining school mathematics in ways that exclude social justice and that create limited subject positions for teachers and students. Further as we attempted to understand teaching mathematics for social justice these limited positions led us to create prescriptive notions of teaching mathematics for social justice.

School Mathematics ≠ Social Justice Mathematics

Our efforts to understand social justice mathematics often resulted in attempts to fit social justice mathematics into the pure/applied binary of abstract mathematics. School mathematics emphasizes the pure side of this binary and social justice mathematics is understood as a response against this. As a result, we tried to find applied examples that we could use that would highlight social justice issues. The work of Gutstein (2006; 2012) offers the most well-known examples of this kind of social justice mathematics; for some mathematics teachers, this approach may be understood as the only way to do social justice mathematics.

However, this social critique style of social justice mathematics is difficult and trying to always fit social justice into the pure/applied binary became an obstacle. Mathematics teachers understand what it means to apply mathematics, but in most cases we do not know *how* to apply it, because we have never been taught how (at least for secondary mathematics). Further the certainty that is emphasized in dominant discourses of mathematics slips into uncertainty as we try to apply what we know. The one exception to this is statistics and most of the potential social justice examples we discussed drew from statistics. However, since mathematics teachers spend a relatively small portion of their year teaching statistics, this avenue provides for, at best, minimal opportunities to do this kind of social justice mathematics.

Further, the separation of statistics from other branches of dominant mathematics illustrates another influence of the dominant discourses of mathematics. All the various branches of mathematics (algebra, geometry, trigonometry, calculus, statistics, etc.) are understood as separate and distinct. If they were understood as overlapping then our

ability to apply statistics could provide a means of bringing these kinds of social justice lessons into other branches of mathematics. This could occur by teaching algebra and geometry together as different, complementary ways of approaching the same issues or statistics as applying algebra to understand real-world data. Further, when applied mathematics is understood as the one and only way to teach mathematics for social justice, teachers may miss other critical dimensions of social justice mathematics such as student identity and classroom and societal power relations.

Teacher Authority Limits Teaching Mathematics for Social Justice

Our understanding of what it means to teach mathematics for social justice was further constrained by dominant discourses of teacher authority. Dominant discourses of teacher authority emphasize teacher control of what happens in the classroom both in terms of content and in terms of student behavior. We generally associate authoritarian teaching with students sitting quietly in rows, working individually, and taking notes. However, even more progressive teaching associated with student-centered classrooms and constructivist teaching typically also emphasize teacher control of content and student behavior, although the means of control are usually less obvious and perceived as gentler.

The mathematics teacher's control of the classroom is likely to be understood as connected to her knowledge of abstract mathematics. This connection grows out of the certainty of abstract mathematics and of the certainty of judgment that accompanies (implicitly White, because of their "lack of race") positions of authority. Teaching mathematics for social justice (even if doing so is understood strictly in terms of social

critique lessons) often requires the teacher to introduce topics and teaching methods that are less certain. These can lead to a feeling, for the teacher, of loss of control of the class (and the accompanying positioning as bad teacher). The fear of loss of control can preempt teachers' attempts to teach for social justice. Even when we come up with social justice mathematics lessons that allow us to maintain control, we are missing broader aspects of social justice, especially identity and power relations. Instead addressing identity and power likely means that we find ways to teach in less certain, less controlling ways on a regular basis, in addition to the necessarily less frequent social, critique lessons. Thus as we try to force social justice lessons into the dominant discourses we can end up missing more frequent opportunities to bring social justice into our teaching.

Individual Responsibility

While we commonly used the discourse of individual responsibility to deflect blame away from teachers (ourselves), there were other times when we took this responsibility on ourselves. The discourse of individual responsibility is linked to discourses of authority and control. Together these discourses create a cycle of blame. In this cycle people in positions of higher authority, by holding their subordinates "responsible," deflect blame from themselves onto whoever is in the next position below. In a school system, from the institutional perspective, the students and their families are often in the lowest position, and, as a result the group ultimately to blame for lack of success. By passing blame we were distracted from educational inequities and the necessary work of understanding social justice mathematics. Instead the discourse of individual responsibility led us to point at different parties as potentially responsible for the lack of social justice mathematics in schools. However, in those relatively few

moments when we stopped passing blame and took "responsibility," taking responsibility often also meant taking a more authoritative stance. This stance left out a respectful and meaningful role for students. Without this subject position for students our efforts to understand social justice mathematics were severally limited, because they ignored the students we are supposed to work with.

Struggling in the Differences of Discourses

The third of my research questions is: How do secondary mathematics teacher candidates merge/manage and challenge the disparate discourses of mathematics and social justice during student teaching in a program that emphasizes preparation for teaching in culturally and linguistically diverse contexts? As shown in my responses to the first two questions, the dominant discourses of mathematics and mathematics education do not fit well with the discourses of social justice mathematics. These discourses frequently eclipsed our efforts to understand and develop the discourses of social justice mathematics. We struggled to bring the discourses together in ways that both made sense within the dominant discourses of mathematics *and* met our ideals for what social justice mathematics could be. However, there were moments when we were able to temporarily break through the limits of dominant discourses; these moments occurred as we played with our understanding of mathematics teaching and by changing how students could be positioned. Importantly there were also times when we embraced the struggles and contradictions of this work, rather than trying to resolve them (Gutiérrez, 2009; 2015).

Playfulness

There were moments when we played with what it means to teach mathematics. These moments created temporary disruptions in the dominant discourses that allowed us to see the dissonances between the dominant discourse of mathematics education and our ideals of a socially just mathematics education. Although we eventually came back to the dominant discourses, the disruptive moments were still instructive. Being playful changed the way that Karl thought about and approached classroom power relations; it changed the way that he viewed and interacted with his students. After Karl brought up this change in our class, discussions of power relations became more regular parts of our class discussions and how I approached the class that I was teaching. I learned to let go of some of my control of the class, allowing the students to develop the discussion, and to not always insist on the last word of a discussion.

Student Subject Positions

Most of the time we were unable to construct subject positions for students that were not more or less passive. Viewing students as passive undermined our ability to imagine socially justice mathematics teaching that did not seem to require an authoritarian teaching style. However, occasionally we understood students beyond this passive/participant binary by including roles such as active, critical, or skeptical. These student roles required that our teacher roles shift in response. Of these moments two stand out. The first came as Lisa suggested that students of color may not trust their White teachers, and that they may be right not to trust them. This comment simultaneously created an intelligently skeptical subject position for students and questioned the credibility (authority) of the efforts of White teachers in teaching students

of color. The point Lisa made caused us to reconsider what it means to teach mathematics for social justice and that we not impose our version of social justice on our students. Instead, if we learn to work with our skeptical students, we may together develop more socially just teaching practices (Margonis, forthcoming)

The importance of breaking out of the passive/participant student binary was highlighted as Esperanza drew on her experiences as an immigrant Latina, ESL student. As she talked about herself as a student, the times when teachers made her angry or frustrated as well as the times teachers showed that they cared, we could see our students as people (not just as students) in our classes. This view of students requires that teachers be responsible *to* their students (as opposed to responsible *for*). This includes a responsibility to listen to and understand their perspectives, and importantly to engage in the challenges of teaching mathematics for social justice by working with our students.

Struggle and Contradiction

As a class we regularly struggled with the multiple contradictions of the dominant discourses of mathematics education and social justice. While the dominant discourses pushed us to resolve those contradictions in a quest for clarity, solutions, or becoming "good" teachers, these resolutions typically ended up returning us to the standard, traditional solutions of the dominant discourses that we were trying to disrupt. To teach mathematics for social justice requires living with and embracing the tensions (Gutiérrez, 2009; 2015; North, 2008) that are created from these contradictions. Not resolving tensions can be uncomfortable and difficult as Esperanza noted in her journals and Lisa illustrated in her final project. However, not resolving these tensions provides time and discursive space for the reimagining of our understanding of what it means to teach for

social justice.

Implications

Mathematics Education

School mathematics is dominated by the discourses of abstract mathematics. These discourses overlap with discourses of Whiteness (e.g., judgment, individual responsibility, authority) to help maintain White privilege throughout the education system. These discourses can be challenged by addressing the binary of pure/applied mathematics, the separation of mathematics into its various branches, and the lack of recognition for alternative conceptions of what mathematics is and what it means to do mathematics. My experiences suggest that playfulness, repositioning students, and embracing struggle (over resolution) can create opportunities to disrupt these discourses.

Mathematics education would significantly benefit from greater emphasis on how to apply the mathematics that teachers learn in college, and especially the mathematics that they will teach to their students. While this emphasis, by itself, will not disrupt the pure/applied binary it will help to challenge the privileged position of pure mathematics, at least for mathematics teachers. The emphasis on pure mathematics is further exacerbated by the separation of mathematics into various branches. This separation causes teacher candidates to miss out on the many links between the branches and how they inform each other. Central to understanding these links between branches is understanding the historical development of mathematics around the world, and how mathematics is *used* both formally and informally around the world, but especially by nonmathematicians. These changes (more applied and more connected mathematics) could facilitate a broader understanding of mathematics for preservice teacher candidates.

In particular, these changes could disrupt the idea of mathematics as certain, acultural and apolitical, while also establishing a basis for alternative views of mathematics.

Mathematicians are not in agreement as to what mathematics is and what it means to do mathematics (Ernest, 1991). Further there is a disconnect between what mathematicians actually do and the view of mathematics that is presented in schools (Boaler & Greeno, 2000). However, in general, mathematics classes do not address these debates and distinctions and instead present a unified, certain view of what mathematics is (Boaler & Greeno, 2000). This view of mathematics is alienating for many students and leads to misconceptions both about what it means to be a mathematician (and who could be) and what it means to do mathematics. Mathematics teachers should be able to draw on their changed mathematics preparation to teach in ways that disrupts dominant notions of what it means to do mathematics. Learning to play with mathematics (i.e., take risks, try things out, and experiment) may serve to break down some of the mystique and reverence that we give to mathematics, even as teachers. However, since these dominant discourses are also prevalent in school mathematics departments, student-teachers will need to be paired with supportive mentors and university supervisors. This support will likely need to continue into at least the beginning portion of the mathematics teacher's career.

Playfulness

Mathematical play seemed to be particularly challenging for these teacher candidates (this was the failed activity in week 3, see Appendix), at least as I approached it. However, play may be an important way to facilitate disruption of the dominant discourses of mathematics as abstract. In my class, in addition to exploring new content

(discourses), suspending judgment and evaluation appeared to open up opportunities for playfulness. Mathematics topics or classes that "level the playing field" (i.e., where everyone comes in as a novice) and establishing noncompetitive activities (to lessen judgment) for these topics may aid mathematics teachers in learning to play with mathematics and thereby be more willing and able to understand nondominant forms of mathematics and to facilitate mathematical play among their students. For example, in my teacher preparation our methods course was a year-long class that combined pedagogy, mathematical history, and number theory. Since none of us in the class had studied number theory the content allowed us to play with mathematical ideas in ways that we had not considered before.

Authority

In planning and teaching my class I made specific choices and plans to facilitate shared decision making and to respond to (be responsible to) the teacher candidates and their needs. I made sure there were times in each class for the teacher candidates to talk about what they were struggling with (in my class, in other classes, or in student teaching) and adjusted what we were doing to meet their needs. This required balancing my goals for the class with theirs. As Karl began studying his own use of authority in his teaching I paid more attention to my own use of authority. I noticed in the process a few ways in which I was maintaining control (while appearing not to do so overtly). For example, I frequently tried to control the direction of the discussion, step in to correct students, or to always have the last word in any discussion. These are all methods of controlling the content of a discussion. While there are times when any of these techniques may be appropriate, I believe I overuse them. As a result, student

contributions and learning are potentially limited. I discussed this use of authority with the teacher candidates and worked on changing the way I approached these situations. While I believe that these efforts provided a less authoritative teaching model to these teacher candidates, the impact was likely less than it could be because of the way mathematics is positioned as the exception by dominant discourses.

Struggle

Finally teacher candidates need time and support to engage in the struggle of bringing mathematics and social justice together. While this requires, at times, that professors not push students to draw conclusions, the dominant discourses still encourage conclusions. As a result professors may need to create activities that are open to exploration and uncover some of the disagreements and debates within mathematics, rather than hiding them. This also requires that professors pay attention to and disrupt students' attempts to draw conclusions, while providing support and encouragement to keep working through ideas—both mathematical and educational.

Mathematics Teacher Education

The above changes in the mathematics curriculum of teacher candidates would form an important component of their preparation. However, there are also changes on the education side that will be useful. Often in classes on multicultural education, special education, classroom management, or assessment, etc. mathematics is constructed, by the instructors and texts, as an exception to the general principles being taught. This likely happens because mathematics teachers tend to have difficulty understanding how the general principles apply to their context, because the examples given traditionally do not

address mathematics, and because many of the professors of these courses do not have the experience with mathematics necessary to confront the idea of mathematics as an exception. Part of creating and imaging what social justice mathematics education could be will be facilitated as we create opportunities for students to play with the ideas they are learning, to think of students as having authority, and to work within the challenges of teaching.

Those of us in teacher education with a mathematical background need to work with our critical colleagues to collapse the view of mathematics as more "pure" (universal, apolitical, acultural, neutral, etc.) than other subject areas. The view of mathematics as more "pure" is part of the dominant discourses that deflect thinking away from social justice. Challenging the view of mathematics as more "pure" includes a more thorough theorization of how mathematics has been culturally and politically shaped, and how mathematics currently *is* both cultural and political. As all of us in teacher education work with teacher candidates, we need to avoid the push to resolution that leads to narrow and prescriptive views of social justice generally and in mathematics specifically (Gutiérrez, 2015). We need to develop, play with, and publish more varied examples of what teaching mathematics for social justice looks like in order to create a broader view of social justice mathematics than that presented in the social critique lessons that have become the most common form of social justice mathematics.

Engaging in social justice work is a struggle. Teacher candidates will need the support of classmates, mentors, and professors to understand and engage in social justice efforts. As Esperanza's experience suggests, teacher candidates of color may need different kinds of support than White teacher candidates. We cannot assume that teacher

candidates of color will automatically understand how to teach for social justice (Montecinos, 2004). As teacher candidates leave the university and enter the teaching field they will likely need continued support to develop the practice of teaching mathematics for social justice. Esperanza and Lisa attempted to do this for themselves by beginning a social justice focused teachers' group that I helped organize and facilitate.

Playfulness

I did not approach this particular class with the intention of creating playful moments. Rather the concept of playfulness came as I studied relevant literature while I was working through my analysis. From that analysis there are certain aspects of activities that seem to facilitate playfulness that I will more consciously engage in future teaching. I believe that my approach to the class, establishing shared authority, helped to create space and willingness to play. The most sustained moment of playfulness came as we were discussing discourses. This playfulness seemed to have been enabled by having teacher candidates work with an unfamiliar topic (discourses). Thus there was less pressure for them to prove themselves to me or to each other.

I also felt less pressure to lead the students to specific conclusions. This reduction in pressure helped me be willing to let the conversation go rather than trying to guide and direct it (through evaluation, IRE cycles) to a specific destination. Karl had also been thinking about the idea of authority prior to this moment (possibly facilitated by the action research focus of the class). These circumstances seemed to create an opportunity for playful moments to emerge. In future classes I will (and have) worked to intentionally create playful moments. It will probably be useful to have some means through language and class culture to let students know when playfulness is encouraged. However, in doing

so it is important that playfulness not become another way to judge or evaluate students (or for them to judge or evaluate each other) as playful or not, since that evaluation would diminish the possibilities for play. I do not suggest that these are the only ways to facilitate playfulness. However, these were conditions that I found helpful.

Authority

In my teaching I tried to balance the ways I used my authority to teach in ways that were respectful of and responsive to my students (i.e., working equitably with them) with what I see as the need for them to develop commitments and capability to teach for social justice with their own students. Thus I made teaching mathematics for social justice a central focus of the class and, at times, required my students to consider how they could address social justice in their teaching and in their action research projects. However, I did not require that they choose explicitly social justice focused topics for their action research projects. I do not know if this was the "right" balance, but I believe it is important to find a balance between these (sometimes) competing demands, and to explain to our students the reasons behind what we do.

This balance will likely need to shift depending on the particular course and the students who are in those courses. For example, it seems to make sense that a multicultural education course could go farther in requiring students to address social justice through content, while a course on "classroom management" would focus more on the ethical treatment of students and issues of teacher authority. I also tried to emphasize to students that, while their projects were graded, I would rather see (and would grade accordingly) a failed attempt at something meaningful and challenging than a successful attempt at something less meaningful. As a result as they chose and explained their

projects, none of the teacher candidates attempted the social critique lessons that I was hoping for, but several of them addressed social justice in nuanced and creative ways. Their efforts pushed me to reconsider what "counts" as social justice mathematics in my mind and to look beyond the social critique lessons that more obviously address social justice.

Struggle

Encouraging students to become comfortable with and hold onto the dissonances between school mathematics and social justice is likely the most challenging aspect of this work. As with playfulness, struggle was not something that I planned to address going into this class. However, as I analyzed and wrote up the data, unresolved struggles seemed to be a particularly productive experience for our class. I believe that my regular focus on and commitment to social justice topics created opportunities for my students to bring up and discuss their own ideas around social justice. Further, in moments when I focused on discussing ideas with them, rather than evaluating, and allowing discussions to develop, rather than (always) guiding them, the students felt more freedom to struggle through their thinking. While some explicit guidance is necessary, especially when ideas are new, students also need time to explore ideas without feeling like they have to get it "right."

I also tried to reflect support of struggle in my grading policies, encouraging students to take on meaningful, but difficult projects, rather than simple, superficial ones. The students reflection journals and my responses to them provided further opportunity for students to think deeply (without rushing conclusions) and I pushed this thinking through the questions I asked them. In hindsight I believe I could also do more to look for

moments when discussions are drawing too quickly to resolutions, to point out when resolution is happening, and to pull the discussion back away from resolution.

Research

Further research is needed to better understand how to prepare mathematics teachers to teach for social justice. While we have important research documenting some aspects of what it means to teach mathematics for social justice, there is minimal research that considers how mathematics teachers may become social justice educators. This study has considered the role of dominant discourses of mathematics and Whiteness in restricting preparation to teach mathematics for social justice, for student teachers. Potential research avenues that connect to this might consider specifically the discourses that operate within university mathematics departments around teacher preparation, the discourses that operate within schools that adopt social justice aims, or how mathematics is constructed as an exception within colleges of education. While I have described some of the overlaps between discourses of mathematics and those of Whiteness, these overlaps may warrant more in depth exploration. Further research could explore the ways these discourses operated within K-12 mathematics departments or how they operate differently at different levels of public education.

Research is needed that links what mathematics teachers do in practice with what they were taught during teacher preparation and their specific teaching context. This research could consider how a teacher preparation course (as I plan to teach in the future) that makes the discourses of mathematics explicit and seeks to disrupt them supports (or does not) mathematics teachers in learning to teach for social justice. Further research is needed to explore in greater depth the similarities and differences in preparing teachers of

color and White teachers to teach mathematics for social justice. Teaching mathematics for social justice has great potential to address the inequities created and maintained by traditional mathematics education. Further research on social justice mathematics can provide additional guidance and support to preparing teachers for this important work.

APPENDIX

SUMMARY OF READINGS AND CLASS SCHEDULE

Table 2 provides a summary of our course schedule, readings, and assignments.

Table 2 Summary of Class Activities

Week	Date	Class Focus	Readings	Assignments
Week 1	1/9/2014	What is AR?	Lytle, S., & Cochran-Smith, M. (1990). Learning from teacher research: A working typology. (pp. 83-85) Price, J. N., & Valli, L. (2005). Preservice Teachers Becoming Agents of Change Pedagogical Implications for Action Research. (pp. 57-60) Zeichner, K. (2001). Educational action research	Math Educational History (Jan. 16); What does it mean to be a teacher? (Jan. 13th, comments by Jan. 17); Reflection Journal
Week 2	1/16/2014	No Class	Alsup, J. (2006) *Teacher Identity Discourses* Preface and Chapter 1. Zeichner, K. (1995). Reflections of a teacher educator working for social change. (pp. 11-13).	Reflection Journal
Week 3	1/23/2014	How does math structure our world?	Watch the following 8ish minute video (http://io9.com/why-dont-the-simpsons-live-in-a-base-8-world-1456312130)	Reflection Journal
Week 4	1/30/2014	Discourse in Math Ed	Gee, J. (2005). An Introduction to Discourse Analysis. Chapters 1-3 Fairclough, N. (2001). *Language and Power.* Chapter 4	Reflection Journal
Week 5	2/6/2014	Mock Interviews	How to Develop a Professional Portfolio. Chapters 1, 4-5.	Reflection Journal
Week 6	2/13/2014	Mathematics & Equity	Gutiérrez, R. (2012). Context Matters: How Should We Conceptualize Equity in Mathematics Education? Secada, W.G. (1989). Agenda Setting, Enlightened Self-Interest, and Equity in Mathematics Education.	Reflection Journal
Week 7	2/20/2014	Choosing a Topic	Price, J. N., & Valli, L. (2005). (pp. 61-66); Rogers, D., Bolick, C. M., Anderson, A., Gordon, E., Manfra, M. M., & Yow, J. (2007). "It's About the Kids": Transforming Teacher-Student Relationships Through Action Research. (pp. 218-221).	Plan and Lit. Review; Reflection Journal
Week 8	2/27/2014	Develop a Plan	Find & read 3 articles related to your topic.	Reflection Journal
Week 9	3/6/2014	Methods	Greenwood, D. J., & Levin, M. (2006). Introduction to action research: Social research for social change. Chapter 6;	Prepare report for next class.; Reflection Journal
Week 10	3/13/2014	Spring Break	None	Reflection Journal
Week 11	3/20/2014	Report & Develop	None	Adjust Plan; Reflection Journal

Table 2 continued

Week	Date	Class Focus	Readings	Assignments
Week 12	3/27/2014	Mathematics & Equity.	Trexler, L. (2013). Adventures of a Beginning Teacher with social justice mathematics. D'Ambrosio, U. Ethnomathematics, the nature of mathematics and mathematics education. Gutstein, E. (2012) Mathematics as a weapon in the struggle. Gutiérrez, R. (2009). Embracing the inherent tensions in teaching mathematics from an equity stance	Adjust Plan; Reflection Journal
Week 13	4/3/2014	Report & Develop	None	Final presentation; Reflection Journal
Week 14	4/10/2014	Presentations	None	Select artifacts Reflection Journal
Week 15	4/17/2014	Portfolios	Utah Teaching Standards	Finish Portfolios

Selected Assignments

Action Research Project (210 points)

Instructions to students: Based on what you have observed in your student teaching and observations so far, you will be asked to identify something you would like to change in your teaching, in your classroom, or at your school. This may be motivated by a particular aspect of teaching that you would like to change or a problem that you have observed. After you have identified what you would like to change you will develop a plan of action based on relevant research, implement this plan, evaluate the results, and report back to our class. Based on our class discussions and your observations you will make adjustments and repeat this cycle. This will result in a total of four trials. This project will take place over a 10-week period and will result in a final paper which is due following your student teaching responsibilities. An opportunity for sharing results will take place in April. Please note that you will be given detailed guidance as you move through each stage.

A. Description of problem statement and your rationale. (2-3 pages). In this problem statement you should describe what the problem is, why you consider it a problem, and your motivation for doing something about it. **(10 points). Due: Jan. 28th on Canvas.** Review a partner's problem statement and give comments before class on Thursday.

B. **Initial Plan** (3-5 pages): Develop an initial plan based on relevant literature. Describe what you plan to do, how your plan relates to relevant literature, anticipated difficulties of your plan, and how you will evaluate the effectiveness of your plan. Make sure to cite all sources using an appropriate

format. **Due Feb. 11th on Canvas.** Review a partner's work before class on Thursday. **(10 points).**

Implementation, data collection, and reflection

A. Implement your plan for a period of about 8 weeks (approximately February 13 – April 17, 2014). You will do this in 2-week chunks. After each 2-week period you will report on how things are going and make adjustments to your plan based on our class discussions, your data collection, and your reflections on your plan. Both the adjustments that you are making and your reflections should be recorded in your class journal (see below). (**Adjustments, 10 points each)**

B. Collect data on issues related to your plan and proposal for assessing the problem. All data should be written up and should focus on your issue of concern. Your data should be compiled into bi-weekly summaries discussing changes, concerns, or weaknesses you have observed. Reports are due every two weeks on the following dates: **Feb. 20, Mar. 6, Mar. 27. (10 points each)**

C. Reflections: For this class you will keep a regular (at least weekly) reflection journal. Journal entries (1 page minimum) should explore your own thoughts about what you are learning in class, potential action research topics, challenges and success of your plan, adjustments to your plan, thoughts about what you are learning from your project, and responses to my comments on your journal. These reflections should especially serve as a place for you to identify and question your own assumptions about teaching mathematics. I

will occasionally give specific journal prompts. You may keep your journal on Canvas or keep it on your own and upload weekly. **Due before each class (50 points total)**

D. Presentation: You will present your action research project in a formal Poster Session. This date is being determined.

Final paper: The final paper should be a compilation of your previous work. It will include a description of the problem and your rationale for addressing it; a comprehensive literature review; your plan of action, including methods and analysis and adjustments that you made along the way; a conclusion of your findings; discussion of the strengths and weaknesses of your project; a discussion of how your findings will influence your future teaching, including your understanding of the value of action research as professional development for teachers. **Due: April 24. (80 points)**

This assignment was a required component of the course. I tried to use it to help the teacher candidates gain an understanding of themselves/teachers as capable of research in order to make positive changes to their teaching and to help them better understand the work they were currently doing. I attempted to guide their attention to social justice issues that were coming up in their weekly written reflections, but ultimately left the choice of topic up to them.

Mathematics Educational History (3-page minimum)

Instructions to students: Describe your experiences with mathematics in schools throughout your life. Think about what you liked and didn't like about what your teachers did, how your own experiences are evident in your teaching, and things that you would

like to do differently from how you were taught. Also include why you decided to become a math teacher. (Due Jan. 16; 20 points).

I asked teacher candidates to complete this assignment early in the semester. My goal in asking them to do this was to prompt a discussion around how mathematics has provided them with various (unearned) privileges throughout their lives. While the discussion went well, I do not have a recording of it. The teacher candidates wrote about and understood the privileges they had received as mathematically capable. We did not address racial aspects of this privilege at that time. In the future I would add a racial component to this discussion.

What Does It Mean to Be a (math) Teacher? (2-page minimum)

Instructions to students: Based on the assigned readings and your own beliefs explain what you think it means to be a math teacher. What kind of teacher would you like to be ideally? What kind of teacher are you now? Why are these things important to you? (Post on Canvas discussion board by Jan. 13; Comment on other posts by Jan. 17; 20 points).

This assignment was also early in the semester. My goal here was to gain a better understanding of what the teacher candidates perceived as mathematics teaching and how they saw themselves as teachers. Because we missed a class (in order to attend the Creating Balance Conference) we did not spend much time discussing this in class. However, the teacher candidates did read and comment on each paper. In the future I would include class discussion on this topic as well.

Selected In-Class Activities

Week 3: Reimagining Mathematics in Our World

Instructions to students: Reimagine time divisions in a day in base 10, instead of the mix of bases currently used. Describe and name your time divisions. How do you think you would experience a day differently using these divisions?

View this video http://io9.com/why-dont-the-simpsons-live-in-a-base-8-world-1456312130 then answer the following questions. Please write your answers down and bring them to class with you. What assumptions are made in the video?

Choose some aspect of human life that you are familiar with (i.e., film, books, art, science, architecture, calendar and time systems, history, economy, etc.) and imagine how that aspect of life would change if it was base 8 (or you may choose some other based as long as it is not 10, 2, or 12). Describe in detail what things would be like. Feel free to explore and play around with ideas even if you aren't quite sure what things would be like.

The purpose of this assignment was to lead students to see some of the ways in which mathematics and mathematical conventions shape our world and how we experience it, even for those who are not mathematical. While their work was mathematically interesting, it did not have the effect I intended. The students were decidedly not playful and their conclusion was that the effect on experience would be minimal. However, as I have done similar activities with other groups the impact has been greater. Key among the changes that I made were to discuss discourses prior to this activity, to ask students to discuss or write about what a minute or an hour feels like currently, and then to discuss what a minute might feel like in their new time system.

Week 4: Discourses in Mathematics Education

Task set up: Diagram a nonstandard way of laying out the buttons on a phone dial. Ask: how do you think people would respond to a phone that lays out the numbers in this arrangement? Use this video http://www.numberphile.com/videos/keypad_layout.html to introduce how the common phone layout has become dominant and now excludes other possibilities. Also has an interesting bit about where we think about 0 in relation to other numbers. Questions for discussion: What are some of the dominant discourse in school mathematics? How did they become dominant? What alternative discourses are available? (10-15 minutes)

Brainstorm different uses of mathematics (first individually then as a whole class). Once the class has generated a list of 10-15 uses of mathematic I will ask them individually to sort them into groups. Once their groups are sorted I will ask the students to rank their groups (according to their own criteria). Then rank them again according to how they think society, or mathematicians, or another group would rank them. We will then compare and discuss their groups and rankings. Why is it that some uses of mathematics have higher/lower status? How is this status maintained through discourse? What are some uses of mathematics that don't fit into any of our categories? Who benefits from those discourses of mathematics that have become dominant? (30-40 minutes)

My purpose in the first portion of this activity (telephone number pad) is to help the teacher candidates understand how dominant discourses can shape our world and make some things (like alternate telephone number pads) unthinkable or absurd. This then became the basis for the discussion of what dominant discourses of mathematics

education are. In future classes I would spend more time developing the discussion about the number pad. This would allow us to uncover some of the assumptions behind the decision to create the number pad as it currently is and how other criteria could have led to different options. The second portion of the activity (various purposes of mathematics) was intended to work with teacher candidates to understand how discourses operate within the field of mathematics to privilege some areas over others. We did not have enough time to do this activity, although I will use it again in future classes.

Week 7: Understanding Mathematics as Political

Opening activity: Is mathematics political? Ask students about school segregation. What do they know about segregation? What is segregation? What is the history of segregation? Where did the problems occur? (7 minutes). Interpret these graphs on school segregation[23]: (5 minutes). What is the relationship between mathematics and politics? How is mathematics political? How is teaching mathematics political? How is teaching (in general terms) political? (7 minutes).

My purpose with this activity was to counter the dominant discourse that segregation is a problem that we have solved and that segregation was/is a primarily southern issue. Additionally I wanted to use this as a starting point for a discussion on how mathematics, and mathematics teaching, is political. The activity was useful. However, the teacher candidates did not have the historical knowledge of segregation that

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[23] Graphs available at http://civilrightsproject.ucla.edu/research/k-12-education/integration-and-diversity/mlk-national/southern-slippage-growing-school-segregation-in-the-most-desegregated-region-of-the-country/hawley-MLK-South-2012.pdf on page 17.

I had expected and, as a result, the discussion did not go as planned. In the future, I will provide more context for the graphs before getting to the discussion itself. Additionally I think the discussion about mathematics and mathematics education as political may work more effectively earlier in the semester and in connection with the teacher candidates' writing about what it means to be a teacher.

Week 13: Thinking About Alternatives to Abstract Mathematics

Discussion of mapping, ethnomathematics, and math. What is the purpose of a map? What information is typically recorded in a map? What other information might also be useful to record? How about a map and the ocean? What are the connections between a map and mathematics? What information would you include in your map, besides literal information, that would help you to navigate to and from different places that you are familiar with. Discuss stick charts as used preWWII in the Marshall Islands; view images online. What is the potential benefit of not sharing (not generalizing) information? How is this like modern mathematics?

The teacher candidates had struggled both with understanding alternatives to dominant mathematics and the idea of mathematics as universal. I brought in the example of the Marshallese stick charts to illustrate a mathematical alternative to dominant mathematics and to show how mathematics can be (and is currently) used to maintain elite status. This activity was rushed and I did not allow the discussion sufficient time to develop. Additionally I think it would work better with multiple nontraditional examples and with examples that are both historical and current. In the future I may assign students various readings from the *Journal of Mathematics and Culture*

(http://nasgem.rpi.edu/pl/journal-mathematics-cultures37) to gain better understanding of various forms of mathematics.

Week 14: Rethinking What it Means to Be a Teacher

During Class: Thinking about the video you watched (Jeff Duncan-Andrade's Note to Educators https://www.youtube.com/watch?v=8z1gwmkgFss), what we have discussed in class, and what you have learned from your own experience. Without referring to your previous discussion of what it means to be a math teacher, write again about: What does it mean to be a math teacher? Write about 1 page. Compare this to what you wrote previously. What has changed? What is the same?

The purpose of this activity was to give the teacher candidates another opportunity at the end of the semester to understand what it means to be a teacher. They also compared this writing to what they had written at the beginning of the semester in order to understand how their thinking had changed. After they each had some time to compare we discussed how their thinking had changed. In the future I would make this a homework assignment in order to provide the students more time to think about their thinking and how it has changed. The in-class format did not allow for this and their writing was more rushed than it had been early in the semester.

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